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Professor G. H. DARWIN, M.A., LL.D., F.R.S., President, in the Chair.

Henry C. Plummer, B.A., The Owens College, Manchester; and Clement Jennings Taylor, Derby Road, Kenilworth, Cape Town,

were balloted for and duly elected Fellows of the Society.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer, from personal knowledge, being appended :—

William Henry Robinson, Offenden, Walsall, Staffordshire (proposed by Sir R. S. Ball).

Sixty-four presents were announced as having been received since the last meeting, including amongst others :—

Photograph of the great refractor of the Potsdam Observatory, presented by the Observatory.

The theory of the Figure of the Earth carried to the second order of small quantities. By G. H. Darwin, F.R.S., Plumian Professor and Fellow of Trinity College, Cambridge; President of the Royal Astronomical Society.

Introduction.

As far as I know, Airy was the first to include quantities of the second order in investigating the theory of the Earth's figure; his paper is dated 1826, and is published in Part III. of the *Philosophical Transactions of the Royal Society* for that year.

He gave the formula for gravity which I have obtained below (§ 6, (40)). Our results would be *literatim* identical but that my e is expressed by $e \div (1-e)$ in his notation, and that I denote by $-f$ the quantity which he wrote as A. He also established equations, equivalent to my (13) and (14), which express the identity of the surfaces of equal density with the level surfaces. He remarked that these may be reduced to the form of differential equations, but he did not give the results, since he found himself unable to solve them, even for an assumed law of internal density. I have succeeded in solving these equations in this paper.

Airy further concluded that the Earth's surface must be depressed below the level of the true ellipsoid in middle latitudes. He gave no numerical estimate of this depression, but expressed the opinion that it must be very small.

In the second volume of his *Höhere Geodäsie*, Dr. Helmert has also investigated the formula for gravity to the second order of small quantities. The expression for gravity which has been compared with the results of pendulum experiments by Dr. Helmert was taken as having no term dependent on the fourth power of the sine of the latitude. The results of the experiments are somewhat irregular, and there was no apparent advantage in the inclusion of such a term; accordingly Dr. Helmert assumed that such a term is actually evanescent, and pointed out that this implies that the Earth's surface is elevated above the true ellipsoid, instead of being depressed below it, in middle latitudes. There can, I think, be no doubt that there should be depression, and it therefore seems as if it would be safer to adopt such a formula as that given below in § 6 (41) in future reductions of pendulum experiments.

In volume xix. (1889, pp. E, 1–84) of the *Annals of the Observatory of Paris* M. Callandreau has carried out an elaborate investigation of the problems considered in this paper. The publication of my work might, indeed, have been unnecessary were it not that my procedure is, as I think, simpler than his, and that my formulæ are presented in a more tractable shape. I have, however, in some respects, as for instance in the numerical

solution of the differential equations, carried the work somewhat further than he has done; but, on the other hand, he has considered several interesting points on which I do not touch. Our two methods differ in detail from first to last, and it would be rather troublesome to compare them from point to point. I have then been satisfied with the knowledge that we are travelling along parallel roads. M. Callandreau has also written a short but valuable note on the same subject in the *Bulletin Astronomique* for 1897. I refer to this paper in § 12.

Lastly Professor Wiechert has published an important memoir on the distribution of masses in the interior of the Earth in the *Transactions* of the Royal Society of Sciences of Göttingen (1897, pp. 221–243). He has there adduced weighty arguments in favour of the hypothesis that the Earth consists of an iron nucleus with a superstratum of rock. He also has taken into account quantities of the second order, and has calculated interesting numerical results corresponding to his theory. I refer to this paper in §§ 10, 12, and in the summary.

The first part of my paper contains the mathematical investigation, and this is followed by a summary and discussion of results.

Mathematical Investigation.

§ 1. *The Moments of Inertia and the Potential of a homogeneous Spheroid.*

The equation to the surface of a homogeneous oblate ellipsoid of revolution of density w whose semi-axes are a and $a(1-e)$ is

$$r^2\left(\frac{\cos^2\vartheta}{(1-e)^2}+\sin^2\theta\right)=a^2,$$

where r is the radius vector and θ the colatitude measured from the axis of revolution.

If the cubes and higher powers of e are neglected, the equation may be written

$$r=a\{1-e\cos^2\vartheta-\tfrac{3}{2}e^2\sin^2\vartheta\cos^2\vartheta\}.$$

Now let us consider a spheroid of which the equation is

$$r=a\{1-e\cos^2\theta+(f-\tfrac{3}{2}e^2)\sin^2\theta\cos^2\theta\},$$

where f is of the same order of magnitude as e^2.

This surface has ellipticity e, and the excess of its radius vector over that of the true ellipsoid is $af\sin^2\theta\cos^2\theta$. The maximum excess occurs in colatitude 45°, and it amounts to $\frac{1}{4}af$.

I now introduce the zonal harmonics—

$$P_2=\tfrac{3}{2}\cos^2\theta-\tfrac{1}{2},\qquad P_4=\tfrac{35}{8}\cos^4\theta-\tfrac{15}{4}\cos^2\theta+\tfrac{3}{8},$$

so that $\cos^2\theta=\frac{2}{3}P_2+\frac{1}{3}$, $\quad\sin^2\vartheta\cos^2\vartheta=-\frac{8}{35}P_4+\frac{2}{21}P_2+\frac{2}{15}$.

Accordingly, the equation to the spheroid may be written

$$r=a\{1-\tfrac{1}{3}e-\tfrac{1}{5}e^2+\tfrac{2}{15}f-\tfrac{2}{3}(e+\tfrac{3}{14}e^2-\tfrac{1}{7}f)P_2+\tfrac{8}{35}(\tfrac{3}{2}e^2-f)P_4\} \quad (1)$$

This form of the equation will be needed hereafter, but for the present it is convenient to regard the body as consisting of a homogeneous ellipsoid of density w, and with semi-axes a and $a(1-e)$, together with the excess above the ellipsoid of a body of which the equation is

$$r=a\{1+\tfrac{2}{15}f+\tfrac{2}{21}fP_2-\tfrac{8}{35}fP_4\} \quad . \quad . \quad . \quad . \quad (2)$$

The developments will only be carried to the order e^2, and therefore we may regard this excess as a layer of surface density

$$wa(\tfrac{2}{15}f+\tfrac{2}{21}fP_2-\tfrac{8}{35}fP_4),$$

distributed over the surface of a sphere of radius a.

The mass of the excess is clearly $4\pi wa^3(\frac{2}{15}f)$, and the mass of the ellipsoid is $\frac{4}{3}\pi wa^3(1-e)$; hence the mass of the spheroid is given by

$$M=\tfrac{4}{3}\pi wa^3(1-e+\tfrac{2}{5}f) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If ρ, θ, ϕ be the polar coordinates of a point whose Cartesian coordinates are x, y, z we have

$$x^2+y^2=\tfrac{2}{3}\rho^2(1-P_2), \quad \begin{cases} x^2+z^2=\tfrac{2}{3}\rho^2(1+\tfrac{1}{2}P_2)+\tfrac{1}{2}\rho^2\sin^2\theta\cos 2\phi. \\ y^2+z^2=\tfrac{2}{3}\rho^2(1+\tfrac{1}{2}P_2)-\tfrac{1}{2}\rho^2\sin^2\theta\cos 2\phi. \end{cases}$$

Now let C', A', denote the moments of inertia of the shell (2) about the axes of z, and of x or of y, and we have

$$\begin{cases} C' \\ A' \end{cases}=\tfrac{2}{3}\iiint w\rho^4 \begin{cases} 1-P_2 \\ 1+\frac{1}{2}P_2 \end{cases} \sin\theta d\theta d\phi.$$

integrated throughout the layer comprised between the surface (2) and the sphere a.

Then, since $\int\rho^4 d\rho=\frac{1}{5}a^5\{\frac{2}{3}f+\frac{10}{21}fP_2-\frac{8}{7}fP_4\}$, and since the integrals over the surface of a sphere of a spherical harmonic, and also of the product of two harmonics of different orders vanish, we have

$$C'=4\pi wa^5\int_0^\pi(\tfrac{2}{15}f-\tfrac{2}{21}fP_2{}^2)\sin\theta d\theta.$$

$$A'=4\pi wa^5\int_0^\pi(\tfrac{2}{15}f+\tfrac{1}{21}fP_2{}^2)\sin\theta d\theta.$$

Also $\int_0^\pi P_2{}^2\sin\theta d\theta=\frac{2}{5}$, and therefore

$$C'=\tfrac{8}{105}\pi wa^5(\tfrac{4}{7}f), \quad A'=\tfrac{8}{105}\pi wa^5(\tfrac{5}{7}f).$$

Now denoting the moments of inertia of the homogeneous ellipsoid by C″, A″, we have

$$C''=\tfrac{8}{15}\pi wa^5(1-e), \quad A''=\tfrac{8}{15}\pi wa^5(1-2e+\tfrac{3}{2}e^2).$$

The sums $C'+C''$, $A'+A''$, give the moments of inertia of the spheroid ; so that

$$\left.\begin{aligned} C=\tfrac{8}{15}\pi wa^5(1-e+\tfrac{4}{7}f), \quad A=\tfrac{8}{15}\pi wa^5(1-2e+\tfrac{3}{2}e^2+\tfrac{5}{7}f), \\ C-A=\tfrac{8}{15}\pi wa^5(e-\tfrac{3}{2}e^2-\tfrac{1}{7}f). \end{aligned}\right\} \quad (4)$$

It is, in the next place, necessary to evaluate the potential of the spheroid both internally and externally, and I will begin by considering the part contributed by the shell defined by (2). It consists of a spherical shell of mass $\frac{8}{15}\pi wa^3f$ and radius a, together with surface density $\frac{2}{21}waf P_2-\frac{8}{35}waf P_4$. Accordingly, if U_e', U_i' denote the external and internal potentials

$$U_e'=\tfrac{8}{15}\pi wa^3\frac{f}{r}+\tfrac{4}{5}\pi\cdot\tfrac{2}{21}waf.\frac{a^4}{r^3}P_2-\tfrac{4}{9}\pi\cdot\tfrac{18}{35}waf.\frac{a^6}{r^5}P_4,$$

$$U_i'=\tfrac{8}{15}\pi wa^2f+\tfrac{4}{5}\pi\cdot\tfrac{2}{21}waf.\frac{r^2}{a}P_2-\tfrac{4}{9}\pi\cdot\tfrac{8}{35}waf.\frac{r^4}{a^3}P_4.$$

We have now to find the external and internal potentials of the ellipsoid, which may be denoted by U_e'', U_i''.

It is well known that the external potential of an ellipsoid of revolution of semi-major axis a, *eccentricity* η, and mass M' is

$$U_e''=\frac{M'}{r}\left\{1-\tfrac{1}{5}\frac{a^2\eta^2}{r^2}P_2+\tfrac{3}{35}\frac{a^4\eta^4}{r^4}P_4\ \ldots\right\}$$

Also if $\eta=\sin\gamma$, the internal potential is

$$U_i''=\frac{3M'\gamma}{\sin\gamma}-\tfrac{3}{4}\frac{M'}{a^3\sin^3\gamma}(\gamma-\sin\gamma\cos\gamma)(x^2+y^2)$$
$$-\tfrac{3}{2}\frac{M'}{a^3\sin^3\gamma}(\tan\gamma-\gamma)z^2.$$

Now $$\tan\gamma-\gamma=\tfrac{1}{3}\eta^3(1+\tfrac{9}{10}\eta^2+\tfrac{45}{56}\eta^4+\ldots),$$

$$\gamma-\sin\gamma\cos\gamma=\tfrac{2}{3}\eta^3(1+\tfrac{3}{10}\eta^2+\tfrac{9}{56}\eta^4+\ldots).$$

Also $$(x^2+y^2)=\tfrac{2}{3}r^2(1-P_2), \quad z^2=\tfrac{2}{3}r^2(\tfrac{1}{2}+P_2).$$

Therefore

$$U_i''=\tfrac{3}{2}\frac{M'}{a}(1+\tfrac{1}{6}\eta^2+\tfrac{3}{40}\eta^4\ldots)-\tfrac{1}{2}\frac{M'r^2}{a^3}(1+\tfrac{1}{2}\eta^2+\tfrac{3}{8}\eta^4+\ldots)$$
$$-\tfrac{1}{5}\frac{M'r^2}{a^3}(\eta^2+1\tfrac{5}{14}\eta^4\ldots)P_2.$$

But $\eta^2=2e(1-\frac{1}{2}e)$ and $M=\frac{4}{3}\pi wa^3(1-e)$, and on substitution in the above formulæ it will be found that

$$U_e''=\tfrac{4}{3}\pi w\frac{a^3}{r}(1-e)-\tfrac{8}{15}\pi w\frac{a^5}{r^3}e(1-\tfrac{3}{2}e)P_2+\tfrac{16}{35}\pi w\frac{a^7}{r^5}e^2P_4,$$

$$U_i''=2\pi wa^2(1-\tfrac{2}{3}e-\tfrac{1}{5}e^2)-\tfrac{2}{3}\pi wr^2-\tfrac{8}{15}\pi wr^2(e+\tfrac{3}{14}e^2)P_2.$$

The external potential U_e of the spheroid is equal to $U_e'+U_e''$, and the internal potential U_i is equal to $U_i'+U_i''$. For application to the problem of the figure of the Earth the potential of rotation must be added, and if ω be the angular velocity of rotation, this potential is $\frac{1}{2}\omega^2r^2\sin^2\theta$ or $\frac{1}{3}\omega^2r^2(1-P_2)$.

Now let V_e, V_i be the external and internal potentials, inclusive of rotation, and we have

$$\left.\begin{aligned} V_e=&\tfrac{4}{3}\pi w\frac{a^3}{r}(1-e+\tfrac{2}{5}f)-\tfrac{8}{15}\pi w\frac{a^5}{r^3}(e-\tfrac{3}{2}e^2-\tfrac{1}{7}f)P_2\\ &+\tfrac{16}{35}\pi w\frac{a^7}{r^5}(e^2-\tfrac{2}{3}f)P_4+\tfrac{1}{3}\omega^2r^2(1-P_2)\\ V_i=&\,2\pi\omega a^2(1-\tfrac{2}{3}e-\tfrac{1}{5}e^2+\tfrac{4}{15}f)-\tfrac{2}{3}\pi wr^2\\ &-\tfrac{8}{15}\pi wr^2(e+\tfrac{9}{14}e^2-\tfrac{1}{7}f)P_2-\tfrac{32}{315}\pi w\frac{r^4}{a^2}fP_4+\tfrac{1}{3}\omega^2r^2(1-P_2) \end{aligned}\right\}\quad(5)$$

The first term of V_i is independent of r, θ and is only inserted in order that V may be a continuous function at the surface of the spheroid.

It will be supposed hereafter that the heterogeneous Earth is built up of layers of density w, bounded externally and internally by spheroids defined by $a+\delta a$, $e+\delta e$, $f+\delta f$ and a, e, f. It is obvious that the potential of such a body may be written down from (5) by replacing each term by an integral.

If a, e, f denote the superficial values of a, e, f we shall have such integrals as

$$\int_0^a w\frac{d}{da}[a^5(e-\tfrac{3}{2}e^2-\tfrac{1}{7}f)]da \text{ and } \int_a^a w\frac{d}{da}[e+\tfrac{9}{14}e^2-\tfrac{1}{7}f]da.$$

For the sake of brevity I shall write these

$$\int_0^a wd[a^5(e-\tfrac{3}{2}e^2-\tfrac{1}{7}f)] \text{ and } \int_a^a wd[e+\tfrac{9}{14}e^2-\tfrac{1}{7}f].$$

It may be well to note here that for the heterogeneous Earth

$$\left.\begin{aligned} M&=\tfrac{4}{3}\pi\int_0^a wd[a^3(1-e+\tfrac{2}{5}f)]\\ C&=\tfrac{8}{15}\pi\int_0^a wd[a^5(1-e+\tfrac{4}{7}f)]\\ A&=\tfrac{8}{15}\pi\int_0^a wd[a^5(1-2e+\tfrac{3}{2}e^2+\tfrac{5}{7}f)]\\ C-A&=\tfrac{8}{15}\pi\int_0^a wd[a^5(e-\tfrac{3}{2}e^2-\tfrac{1}{7}f)] \end{aligned}\right\}\quad\ldots\quad(6)$$

§ 27. *Heterogeneous Planet. The surfaces of equal density are level surfaces.*

It will now be supposed that the substance of which the heterogeneous planet is formed is plastic enough to allow the surfaces of equal density to be level surfaces.

It is necessary to write down the potential of the planet at any internal point, and for this purpose I find it better to introduce a new parameter in place of the ellipticity e. This parameter is h, defined by

$$h=e-\tfrac{2}{1}\tfrac{5}{4}e^2-\tfrac{1}{7}f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

I do not quite understand why this substitution should lead to simplification, but I may remark that $h=e-2e^2-\frac{1}{7}(f-\frac{3}{2}e^2)$, and that $f-\frac{3}{2}e^2$ is the coefficient of $\sin^2\theta\cos^2\theta$ in the equation to the spheroid. Thus the existence of the fraction $\frac{2}{1}\frac{5}{4}$ is in some sense explained.

It will be found that the equation (1) to the stratum a, in terms of this parameter, becomes

$$r=a\{1-\tfrac{1}{3}h-\tfrac{1}{2}\tfrac{6}{1}\tfrac{7}{0}h^2+\tfrac{3}{3}\tfrac{}{5}f-\tfrac{2}{3}h(1+2h)P_2+\tfrac{8}{3}\tfrac{}{5}(\tfrac{3}{2}h^2-f)P_4\} \quad . \quad (8$$

$$\left.\begin{aligned} \text{Also} \qquad C&=\tfrac{8}{1}\tfrac{}{5}\pi\int_0^a wd[a^5(1-h-\tfrac{2}{1}\tfrac{5}{4}h^2+\tfrac{3}{7}f)] \\ C-A&=\tfrac{8}{1}\tfrac{}{5}\pi\int_0^a wd[a^5(h+\tfrac{2}{7}h^2)] \end{aligned}\right\} \quad . \quad . \quad (9)$$

Let us now for brevity write

$$\left.\begin{aligned} S_0&=\int_0^a wd[a^3(1-e+\tfrac{2}{5}f)] &&=\int_0^a wd[a^3(1-h-\tfrac{2}{1}\tfrac{5}{4}h^2+\tfrac{2}{3}\tfrac{}{5}f)] \\ S_2&=\int_0^a wd[a^5(e-\tfrac{3}{2}e^2-\tfrac{1}{7}f)] &&=\int_0^a wd[a^5(h+\tfrac{2}{7}h^2)] \\ S_4&=\int_0^a wd[a^7(e^2-\tfrac{2}{9}f)] &&=\int_0^a wd[a^7(h^2-\tfrac{2}{9}f)] \\ T_2&=\int_a^a wd[e+\tfrac{2}{1}\tfrac{}{4}e^2-\tfrac{1}{7}f] &&=\int_a^a wd[h+\tfrac{1}{7}h^2] \\ T_4& &&=\int_a^a wd[\frac{f}{a^2}] \end{aligned}\right\} \quad (10)$$

The terms of the second order in S_0 are not required, so that, in fact, I take $S_0=\int_0^a wd[a^3(1-h)]$.

The formulæ (5) and (10) enable us to write down the potential at any internal point as follows :—

$$\frac{3V}{4\pi}=\frac{S_0}{r}-\tfrac{2}{5}\frac{S_2P_2}{r^3}+\tfrac{1}{3}\tfrac{2}{5}\frac{S_4P_4}{r^5}-\tfrac{2}{5}r^2T_2P_2-\tfrac{8}{1}\tfrac{}{05}r^4T_4P_4+\frac{\omega^2}{4\pi}r^2(1-P_2)$$
$$. \quad . \quad . \quad . \quad (11)$$

If it be assumed that the equipotential surfaces are also surfaces of equal density, the equation $V=$ constant must be reducible to the form (8). Hence, if in (11) we attribute to r the value (8), the coefficients of P_2 and of P_4 must vanish. In effecting this substitution we attribute to r its full value in the term of the lowest order, namely S_0/r; in terms of the first order, namely those involving S_2, T_2, and ω^2, we may put $a(1-\frac{1}{3}h-\frac{2}{3}hP_2)$ for r; and in the terms of the second order, namely, those involving S_4 and T_4, we simply put a for r. A consideration of (11) shows that there are several functions of r, P_2, P_4, which will have to be evaluated, and it is obvious that the expressions in question, when developed to the required order, will involve P_2^2. But

$$P_2^2=\tfrac{1}{5}+\tfrac{2}{7}P_2+\tfrac{18}{35}P_4,$$

and it will be found by aid of this formula that, to the required order of approximation,

$$\left.\begin{aligned}
&\frac{a}{r}=1+\tfrac{1}{3}h+\tfrac{20}{21}h^2-\tfrac{3}{35}f+\tfrac{2}{3}h(1+\tfrac{20}{7}h)P_2+\tfrac{8}{35}(f-\tfrac{1}{2}h^2)P_4\\
&\frac{a^3}{r^3}P_2=\tfrac{2}{5}h+(1+\tfrac{11}{7}h)P_2-\tfrac{24}{35}hP_4\\
&\frac{r^2}{a^2}P_2=-\tfrac{4}{15}h+(1-\tfrac{22}{21}h)P_2-\tfrac{24}{35}hP_4\\
&\frac{r^2}{a^2}(1-P_2)=1-\tfrac{2}{5}h-(1+\tfrac{2}{7}h)P_2+\tfrac{24}{35}hP_4
\end{aligned}\right\}\quad(12)$$

By aid of (12) it may be shown that the conditions that the level surfaces shall be surfaces of equal density are

$$\frac{S_0}{a}(h+\tfrac{20}{7}h^2)-\tfrac{3}{5}\frac{S_2}{a^3}(1+\tfrac{11}{7}h)-\tfrac{3}{5}a^2T_2(1-\tfrac{22}{21}h)-\frac{3\omega^2}{8\pi}a^2(1+\tfrac{2}{7}h)=0 \quad . \quad . \quad (13)$$

$$\frac{S_0}{a}(f-\tfrac{1}{2}h^2)-\tfrac{9}{5}\frac{S_2}{a^3}h+\tfrac{3}{2}\frac{S_4}{a^5}+\tfrac{6}{5}a^2T_2h-\tfrac{1}{3}a^4T_4+\frac{3\omega^2}{4\pi}a^2h=0 \quad . \quad . \quad (14)$$

Since ω^2a^3/S_0 is a quantity of the same order as h, (13) involves terms of two orders, but (14) consists entirely of terms of the second order.

If the terms of the second order be omitted, (13) becomes

$$\frac{S_0}{a}h-\tfrac{3}{5}\frac{S_2}{a^3}-\tfrac{3}{5}a^2T_2-\frac{3\omega^2a^2}{8\pi}=0 \quad . \quad . \quad . \quad . \quad . \quad (15)$$

This may be used for the elimination of T_2 and of ω^2 from (14); for multiplying it by $2h$ and adding it to (14) we have

$$\frac{S_0}{a}(f+\tfrac{3}{2}h^2)-3\frac{S_2}{a^3}h+\tfrac{3}{2}\frac{S_4}{a^5}-\tfrac{1}{3}a^4T_4=0 \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The conditions for the identity of the two surfaces are then (13) and (16).

In order to obtain the differential equations to be satisfied by h and f, it is necessary to eliminate the S and T integrals by differentiation, and the results may be much simplified by the use of the approximate form (15) of (13), and of its derivatives.

Our first task is, then, to pursue the approximate equation (15).

The equation (15) may be written in the form

$$\frac{S_0}{a^3}h-\tfrac{3}{5}\frac{S_2}{a^5}-\tfrac{3}{5}T_2-\frac{3\omega^2}{8\pi}=0.$$

To the same degree of approximation we have $S_0=3\int_0^a wa^2da$, so that $\frac{dS_0}{da}=S_0'=3wa^2$. It follows, therefore, that

$$w=\tfrac{1}{3}\frac{S_0'}{a^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

On differentiating our equation and using (17) we find

$$S_2=S_0a^3(\frac{h}{a}-\tfrac{1}{3}\frac{dh}{da}). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Differentiating again, and effecting some reductions, we have

$$S_0\frac{d^2h}{da^2}=\frac{6S_0}{a^2}h-2S_0'(\frac{dh}{da}+\frac{h}{a}). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

It would be easy, by means of (18), to eliminate S_2 from the terms of the second order in (13), but I prefer to postpone that elimination for the present; we can, however, at once eliminate it from (16). Effecting this elimination, and repeating (13), our conditions are

$$\frac{S_0}{a}(h+\tfrac{2}{7}h^2)-\tfrac{3}{5}\frac{S_2}{a^3}(1+\tfrac{1}{7}h)-\tfrac{3}{5}a^2T_2(1-\tfrac{2}{7}h)-\frac{3\omega^2}{8\pi}a^2(\tfrac{1}{3}+\tfrac{2}{7}h)=0$$

$$\frac{S_0}{a^5}(f-\tfrac{3}{2}h^2+ah\frac{dh}{da})+\tfrac{3}{2}\frac{S_4}{a^9}-\tfrac{1}{3}T_4=0 \quad . \quad . \quad . \quad (20)$$

The equation (19) has not been used yet, but it will prove useful hereafter.

§ 3. *The Differential Equation for f.*

By differentiating the second of (20) and using (17) and (19), I find

$$S_0a^5\Big(\frac{df}{da}-5\frac{f}{a}+\tfrac{27}{2}\frac{h^2}{a}-7h\frac{dh}{da}+a(\frac{dh}{da})^2\Big)-\tfrac{27}{2}S_4=0 \quad . \quad . \quad . \quad (21)$$

Differentiating again, and again using (17) and (19), I find

$$S_0a^5\Big(\frac{d^2f}{da^2}-20\frac{f}{a^2}+12\frac{h^2}{a^2}+4\frac{hdh}{ada}-\Big(\frac{dh}{da}\Big)^2\Big)$$

$$+S_0'a^62\Big(\frac{df}{ada}+\frac{2f}{a^2}-4\frac{h^2}{a^2}-6\frac{hdh}{ada}-3\Big(\frac{dh}{da}\Big)^2\Big)=0 \quad . \quad . \quad (22)$$

I now introduce a new symbol w_0, which is to denote the mean density of all the matter lying inside the spheroid defined by a; then

$$S_0=\int_0^a w da^3=w_0a^3$$

$$S_0'=3wa^2.$$

Thus $\dfrac{S_0'}{S_0}=3\dfrac{w}{a\,w_0}$.

In obtaining this result the ellipticity of the spheroid has been neglected, but for the present this approximation suffices, and (22) is then easily reducible to the form

$$\frac{d^2f}{da^2}+6\frac{w}{w_0}\frac{df}{ada}-(20-6\frac{w}{w_0})\frac{f}{a^2}+12(1-\frac{w}{w_0})\frac{h^2}{a^2}+(4-18\frac{w}{w_0})\frac{hdh}{ada}$$

$$-(1+9\frac{w}{w_0})(\frac{dh}{da})^2=0 \quad . \quad (23)$$

This is the differential equation for f, and I shall in § 9 solve it on the assumption of a certain law of internal density of the Earth.

§4. *The Differential Equation for h.*

I now return to the first of (20), and divide it by $a^2(1-\frac{2}{21}h)$, so that it becomes

$$\frac{S_0}{a^3}(h+\tfrac{82}{21}h^2)-\tfrac{3}{5}\frac{S_2}{a^5}(1+\tfrac{5}{21}h)-\tfrac{3}{5}T_2'-\frac{3\omega^2}{8\pi}(1+\tfrac{4}{3}h)=0 \quad . \quad (24)$$

As a preliminary to the differentiation which will eliminate the T_2 integral I indicate certain transformations.

We have $$S_0=\int_0^a w d[a^3(1-h)]$$

$$S_0'=3wa^2(1-h-\tfrac{1}{3}a\frac{dh}{da}).$$

Hence $$w=\frac{S_0'}{3a^2}(1+h+\tfrac{1}{3}a\frac{dh}{da}) \quad . \quad . \quad . \quad . \quad . \quad (25)$$

If S_2, T_2 as defined in (10) be differentiated, and if the value of w as given in (25) be used, it will be found that

$$\left.\begin{aligned} 3\frac{dS_2}{da}&=\frac{S_0'}{a^2}\Big(5h+a\frac{dh}{da}+\tfrac{45}{7}h^2+\tfrac{68}{21}ah\frac{dh}{da}+\tfrac{1}{3}\Big(a\frac{dh}{da}\Big)^2\Big)\\ \text{and}\\ 3\Big(1+\tfrac{55}{21}h\Big)\frac{dS_2}{da}&=\frac{S_0'}{a^2}\Big(5h+a\frac{dh}{da}+\tfrac{410}{21}h^2+\tfrac{41}{7}ah\frac{dh}{da}+\tfrac{1}{3}\Big(a\frac{dh}{da}\Big)^2\Big)\\ 3\frac{dT_2}{da}&=-\frac{S_0'}{a^2}\Big(\frac{dh}{da}+\tfrac{41}{7}h\frac{dh}{da}+\tfrac{1}{3}a\Big(\frac{dh}{da}\Big)^2\Big)\end{aligned}\right\}(26)$$

On differentiating (24) and using (26) and (18) it will be found that

$$S_2+S_0a^3\Big(-\frac{h}{a}+\tfrac{1}{3}\frac{dh}{da}-\tfrac{9}{7}\frac{h^2}{a}+\tfrac{76}{63}h\frac{dh}{da}+\tfrac{113}{63}a\Big(\frac{dh}{da}\Big)^2\Big)-\frac{\omega^2a^6}{6\pi}\frac{dh}{da}=0. \quad (27)$$

On differentiating (27) and again using (26) and (19) in the small terms it will be found that

$$\begin{aligned} &S_0\Big(\frac{d^2h}{da^2}-6\frac{h}{a^2}+14\frac{h^2}{a^2}+\tfrac{66}{7}\frac{hdh}{ada}+\tfrac{40}{7}\Big(\frac{dh}{da}\Big)^2\Big)\\ &+S_0'\Big(2\frac{dh}{da}+2\frac{h}{a}-\tfrac{14}{3}\frac{h^2}{a}-\tfrac{522}{21}ah\frac{dh}{da}-\tfrac{226}{21}a\Big(\frac{dh}{da}\Big)^2\Big)\\ &\qquad-\frac{3\omega^2a^2}{\pi S_0}(S_0-\tfrac{1}{3}S_0'a)\Big(\frac{h}{a}+\frac{dh}{da}\Big)=0. \quad (28)\end{aligned}$$

Since $S_0=w_0a^3(1-h)$, it follows from (25) that

$$\frac{S_0'}{S_0}=\frac{3}{a}\frac{w}{w_0}\Big(1-\tfrac{1}{3}a\frac{dh}{da}\Big) \quad (29)$$

I shall later denote by m the ratio of equatorial centrifugal force to equatorial gravity, and by an extension of this notation I will now write

$$m=\frac{\omega^2a^3}{\frac{4}{3}\pi S_0} \quad (30)$$

By means of (29) and (30) equation (28) becomes

$$\begin{aligned} &\frac{d^2h}{da^2}+6\frac{w}{w_0}\frac{dh}{ada}-\Big(1-\frac{w}{w_0}\Big)\Big[6\frac{h}{a^2}-14\frac{h^2}{a^2}-\tfrac{66}{7}\frac{hdh}{ada}-\tfrac{40}{7}\Big(\frac{dh}{da}\Big)^2\\ &\qquad+4m\Big(\frac{h}{a^2}+\frac{dh}{ada}\Big)\Big]=0 \quad (31)\end{aligned}$$

This is the differential equation to be satisfied by h. If the terms of the second order be omitted it is the equation for the

ellipticity of internal strata as given in any treatise on the theory of the figure of the Earth.

The transformation by which h was substituted for e has enabled us to obtain an equation for h in which f is not involved. Moreover h has been chosen as such a function of e that all the latter terms are multiplied by the simple factor $1-w/w_0$, instead of by various more complex functions of the density, as was the case in the differential equation for f.

§ 5. *Radau's form of the Differential Equation of the Ellipticity.*

I shall now include the terms of the second order in M. Radau's very remarkable transformation of the equation of the last section.

Since $$w_0a^3(1-h)=\int_0^a w d[a^3(1-h)],$$

it follows that

$$\frac{w}{w_0}=1+\tfrac{1}{3}(1+\tfrac{1}{3}a\frac{dh}{da})\frac{adw_0}{w_0da} \quad . \quad . \quad . \quad . \quad . \quad (32)$$

The equation (31) may therefore be written

$$\frac{d^2h}{da^2}+6\frac{dh}{ada}+2\frac{adw_0}{w_0da}(\frac{dh}{ada}+\frac{h}{a^2})$$
$$-\tfrac{1}{5}\frac{adw_0}{w_0da}\{14\frac{h^2}{a^2}+\tfrac{5}{7}\tfrac{2}{}\frac{hdh}{ada}+\tfrac{2}{7}\tfrac{6}{}(\frac{dh}{da})^2-4m(\frac{dh}{ada}+\frac{h}{a^2})\}=0.$$

The last term here includes all the terms of the second order.

If we write $$\eta=\frac{adh}{hda},$$

$$\frac{d^2h}{da^2}=\frac{h}{a^2}(a\frac{d\eta}{da}-\eta+\eta^2), \quad \frac{dh}{ada}=\frac{h}{a^2}\eta.$$

Thus the equation becomes

$$a\frac{d\eta}{da}+5\eta+\eta^2+2\frac{adw_0}{w_0da}(1+\eta)$$
$$-\tfrac{4}{2}\tfrac{}{1}\frac{adw_0}{w_0da}h\{18+\tfrac{1}{2}\tfrac{3}{}(1+\eta)^2-7\frac{m}{h}(1+\eta)\}=0.$$

Now it is easy to prove that

$$a\frac{d\eta}{da}=\frac{2\sqrt{(1+\eta)}}{w_0a^4}\frac{d}{da}[w_0a^5\sqrt{(1+\eta)}]-2(5+\frac{adw_0}{w_0da})(1+\eta).$$

Therefore the equation may be written

$$\frac{2\sqrt{(1+\eta)}}{w_0 a^4}\frac{d}{da}[w_0 a^5\sqrt{(1+\eta)}]=10(1+\tfrac{1}{2}\eta-\tfrac{1}{10}\eta^2)$$
$$+\tfrac{4}{21}\frac{a dw_0}{w_0 da}h\{18+\tfrac{13}{2}(1+\eta)^2-7\frac{m}{h}(1+\eta)\}.$$

In the terms of the second order we may put $\frac{a dw_0}{w_0 da}$ equal to $-3(1-\frac{w}{w_0})$; therefore

$$\frac{d}{da}[w_0 a^5\sqrt{(1+\eta)}]=\frac{5w_0 a^4}{\sqrt{(1+\eta)}}\{1+\tfrac{1}{2}\eta-\tfrac{1}{10}\eta^2$$
$$-\tfrac{2}{35}h(1-\frac{w}{w_0})[18+\tfrac{13}{2}(1+\eta)^2-7\frac{m}{h}(1+\eta)]\} \quad . \quad (33)$$

This is M. Radau's equation, with the inclusion of the terms of the second order. These terms have been determined by M. Callandreau, but the form in which he gives them appears to me more complicated than the above.

Let us consider the function within { } on the right-hand side of the equation (33), and in the first place omitting the terms of the second order consider the function $\frac{(1+\frac{1}{2}\eta-\frac{1}{10}\eta^2)}{\sqrt{(1+\eta)}}$. It is equal to unity when η is zero, rises to a maximum of 1·00074 when $\eta=\frac{1}{3}$, and only falls to ·8 when $\eta=3$. Now it has been proved that η is necessarily less than 3, and is positive.* In all the cases which are likely to prove of practical interest η is very much less than 3. In the case of the Earth, for example, η is equal to about ·56 at the surface, and vanishes at the centre. Now when $\eta=$·56 this function is equal to ·99971. Therefore between the centre and the surface it rises from unity to 1·00074 and then falls to ·99971. It is obvious that any kind of average value of the function, estimated over the range from centre to surface, can at most differ from unity in the fourth place of decimals. When the terms of the second order are included I think the average value is yet nearer unity than when they are omitted, for in such a case as that of the Earth $(1+\frac{1}{2}\eta-\frac{1}{10}\eta^2)\div\sqrt{(1+\eta)}$ is greater than unity throughout the greater part of its range, and, although I cannot prove it absolutely, I believe that $18+\frac{13}{2}(1+\eta)^2-7\frac{m}{h}(1+\eta)$ or $18-\frac{49}{2}\frac{m^2}{h^2}+\frac{13}{2}(1+\eta-\frac{7}{13}\frac{m}{h})^2$ is always positive. If this is the case the whole function is on the average nearer unity than when the terms of the second order are omitted.

* Professor Helmert tells me that he doubts the universal validity of the proof of this. I have to thank him for valuable information given me while writing this paper.

Supposing, then, that $1+\lambda$ denotes a proper mean value of the function

$$\{1+\tfrac{1}{2}\eta-\tfrac{1}{16}\eta^2-\tfrac{2}{35}h(1-\frac{w}{w_0})[\tfrac{18}{5}+\tfrac{12}{5}(1+\eta)^2-7\frac{m}{h}(1+\eta)]\}(1+\eta)^{-1},$$

estimated over the whole range from centre to surface, we must have

$$w_0a^5\sqrt{(1+\eta_1)}=5(1+\lambda)\int_0^a w_0a^4da \quad . \quad . \quad . \quad . \quad (34)$$

where w_0, a, η_1 denote the superficial values of those quantities; and we may feel sure that $1+\lambda$ will not differ from unity until we come to at least the fourth place of decimals.

I shall in § 12 attempt to use this remarkable result for evaluating the ellipticity of the Earth's surface from the Precessional Constant.

§ 6. *Gravity at the Earth's Surface.*

In order to render this presentation of the theory of the Earth's figure more complete I shall now go on to find the theoretical expression for gravity, although the same investigation is to be found in various other places.

At the surface of the planet the integrals T_2, T_4 vanish, and the potential (11) becomes

$$\frac{3V}{4\pi}=\frac{S_0}{r}-\frac{2}{5}\frac{S_2P_2}{r^3}+\frac{12}{35}\frac{S_4P_4}{r^5}+\frac{\omega^2r^2}{4\pi}(1-P_2).$$

In this section, as elsewhere, the superficial values of the various quantities are denoted by Roman in place of Italic type.

Through the vanishing of T_2, T_4 the equations (20), which denote that the surface is a level surface, become

$$\frac{S_0}{a}(h+\tfrac{2}{7}h^2)-\tfrac{3}{5}\frac{S_2}{a^3}(1+\tfrac{11}{7}h)-\frac{3\omega^2}{8\pi}a^2(1+\tfrac{2}{7}h)=0$$

$$\frac{S_0}{a}(f+\tfrac{3}{2}h^2)-3\frac{S_2}{a^3}h+\tfrac{3}{2}\frac{S_4}{a^5}=0$$

It must be observed that the mass M of the spheroid is equal to $\frac{4}{3}\pi S_0$; and we deduce

$$\left.\begin{aligned} S_2&=\frac{5Ma^2}{4\pi}\{h+\tfrac{2}{7}h^2-\tfrac{1}{2}\frac{\omega^2a^3}{M}(1-\tfrac{2}{7}h)\}\\ S_4&=\frac{Ma^4}{2\pi}\{\tfrac{3}{2}h^2-f-\tfrac{5}{2}\frac{\omega^2a^3}{M}h\}\end{aligned}\right\} \quad . \quad . \quad . \quad (35)$$

The potential may therefore be written

$$V=M\Big[\frac{1}{r}-\frac{2}{3}\frac{a^2}{r^3}\{h+\frac{2}{7}h^2-\frac{1}{2}\frac{\omega^2a^3}{M}(1-\frac{2}{7}h)\}P_2$$

$$+\frac{8}{35}\frac{a^4}{r^5}\{\frac{7}{2}h^2-f-\frac{5}{2}\frac{\omega^2a^3}{M}h\}P_4+\frac{1}{3}\frac{\omega^2a^3}{M}\frac{r^2}{a^3}(1-P_2)\Big].$$

At the equator $r=a$, $P_2=-\frac{1}{2}$, $P_4=\frac{3}{8}$, and equatorial gravity, say g_e is equal to $-dV/dr$.

It is easy then by differentiation to show that

$$g_e=\frac{M}{a^2}\Big[1+h+\frac{3}{14}h^2-\frac{3}{7}f-\frac{3}{2}\frac{\omega^2a^3}{M}(1+\frac{2}{7}h)\Big] \quad . \quad . \quad (36)$$

Now let m denote the ratio of equatorial centrifugal force to equatorial gravity, so that

$$m=\frac{\omega^2a}{g_e}$$

It follows that

$$\frac{\omega^2a^3}{M}=m\Big[1+h+\frac{3}{14}h^2-\frac{3}{7}f-\frac{3}{2}\frac{\omega^2a^3}{M}(1+\frac{2}{7}h)\Big]$$

$$=m\Big[1+h-\frac{3}{2}m+\frac{3}{14}h^2-\frac{9}{14}mh+\frac{9}{4}m^2-\frac{3}{7}f\Big] \quad . \quad . \quad (36)$$

The function which naturally arises in the consideration of figures of equilibrium of rotating fluid is the ratio of ω^2 to the density. But

$$M=\frac{4}{3}\pi w_0a^3(1-h-\frac{2}{14}h^2+\frac{2}{35}f);$$

hence

$$\frac{3}{4\pi}\frac{\omega^2}{w_0}=m\Big[1-\frac{3}{2}m+\frac{9}{4}m^2-\frac{3}{7}mh-\frac{6}{35}f\Big].$$

If m were an ideally perfect parameter in which to express our results, h and f should have entirely disappeared from this equation, since m should only depend on ω^2 and w_0, and should be perfectly independent of the figure of the planet.

However, for our present purpose it suffices to take

$$\frac{\omega^2a^3}{M}=m(1+h-\frac{3}{2}m), \text{ or } \frac{3\omega^2}{4\pi w_0}=m(1-\frac{3}{2}m) \quad . \quad . \quad (36)$$

To this order there is no objection to the use of m, and I bow to custom in continuing to use it.

The potential of the planet may now be written

$$V=M\Big[\frac{1}{r}-\frac{2}{3}\frac{a^2}{r^3}(h-\frac{1}{2}m+\frac{2}{7}h^2+\frac{1}{7}mh+\frac{3}{4}m^2)P_2$$

$$+\frac{8}{35}\frac{a^4}{r^5}(\frac{7}{2}h^2-\frac{5}{2}mh-f)P_4+\frac{1}{3}m(1+h-\frac{3}{2}m)\frac{r^2}{a^3}(1-P_2)\Big] \quad (37)$$

It will perhaps be more convenient here to reintroduce the ellipticity instead of h. I observe, then, that $h+\frac{2}{7}h^2$ is equal to $e-\frac{1}{2}e^2-\frac{1}{7}f$, and that throughout the rest of the expression e may be written for h.

The quantity δ used by Dr. Helmert in his *Geodesy* (vol. ii. pp. 77–85) is the same as my $\frac{7}{2}e^2-\frac{5}{2}me-f$, and my results will be found to agree with his.

We have, then,

$$\left.\begin{aligned} V=M\Big[&\frac{1}{r}-\frac{2}{3}\frac{a^2}{r^3}(e-\tfrac{1}{2}m-\tfrac{1}{2}e^2+\tfrac{1}{7}me+\tfrac{3}{4}m^2-\tfrac{1}{7}f)P_2 \\ &+\tfrac{8}{35}\frac{a^4}{r^5}(\tfrac{7}{2}e^2-\tfrac{5}{2}me-f)P_4+\tfrac{1}{3}m(1+e-\tfrac{3}{2}m)\frac{r^2}{a^3}(1-P_2)\Big] \\ g_e=\frac{M}{a^2}&[1+e-\tfrac{3}{2}m+e^2-\tfrac{9}{14}me+\tfrac{3}{4}m^2-\tfrac{4}{7}f] \end{aligned}\right\}. \quad (38)$$

Clairaut's ratio is that of the excess of polar above equatorial gravity to equatorial gravity. I follow Dr. Helmert in denoting this ratio by $\mathfrak{b}$; so that

$$\mathfrak{b}=\frac{g_p-g_e}{g_e}.$$

At the pole $r=a(1-e)$, $P_2=P_4=1$. If $-dV/dr$ be found and these values introduced, we get

$$g_p=\frac{M}{a^2}(1+m+\tfrac{5}{7}me-\tfrac{3}{2}m^2-\tfrac{9}{7}f).$$

Then $$g_p-g_e=\frac{M}{a^2}(\tfrac{5}{2}m-e-e^2+\tfrac{39}{14}me-\tfrac{15}{4}m^2-\tfrac{9}{7}f);$$

and since to the first order $g_e=\frac{M}{a^2}(1+e-\frac{3}{2}m)$, we have

$$\mathfrak{b}=\tfrac{5}{2}m-e-\tfrac{17}{14}me-\tfrac{2}{7}f. \quad . \quad . \quad . \quad . \quad (39)$$

The form of the potential shows that the general expression for gravity must be

$$g=g_e[1+\mathfrak{b}\cos^2\theta+\alpha\sin^2\theta\cos^2\theta],$$

where it remains to determine α.

This may be written

$$g=g_e[1+\tfrac{1}{3}\mathfrak{b}+\tfrac{2}{105}\alpha+(\tfrac{2}{3}\mathfrak{b}+\tfrac{2}{21}\alpha)P_2-\tfrac{8}{35}\alpha P_4].$$

The value of α is then determinable by finding the coefficient of P_4 in the expression for g.

Now $g^2=\left(\frac{dV}{dr}\right)^2+\left(\frac{dV}{rd\theta}\right)^2$, where after differentiation the value (1) is attributed to r.

Suppose that $-\frac{dV}{dr}=\frac{M}{a^2}[G_0+G_1+G_2]$, $\frac{dV}{rd\theta}=\frac{M}{a^2}H_1$, where the suffixes denote the orders of the several terms; then it is easy to prove that $g=\frac{M}{a^2}[G_0+G_1+G_2+\frac{1}{2}\frac{H_1^2}{G_0}]$.

Since G_0 is equal to unity as far as the order zero,

$$g=-\frac{dV}{dr}+\frac{M}{a^2}(\tfrac{1}{2}H_1^2).$$

In order to find H_1 the terms of the second order in (38) are to be dropped, and we find

$$\frac{dV}{rd\vartheta}=\frac{M}{a^2}.2e\sin\theta\cos\vartheta.$$

Therefore $H_1=2e\sin\theta\cos\vartheta$,

and $\frac{1}{2}H_1^2=2e^2\sin^2\theta\cos^2\theta$

$=2e^2(\frac{2}{15}+\frac{2}{21}P_2-\frac{8}{35}P_4)$.

We are only concerned with the term in P_4, and this portion of the transverse component of gravity is $-\frac{16}{35}\frac{M}{a_2}e^2P_4$.

It is next required to find the term in P_4 in $-\frac{dV}{dr}$. After the differentiation of V we shall require to determine the term in P_4 in the following functions, namely, $\frac{a^2}{r^2}$, $\frac{a^4}{r^4}P_2$, $\frac{a^6}{r^6}P_4$, $\frac{r}{a}(1-P_2)$.

Now $\frac{a^2}{r^2}=-\frac{16}{35}(\frac{3}{2}e^2-f)P_4+3(\frac{2}{3}e)^2P_2^2+\ldots=\frac{16}{35}fP_4+\ldots$

$\frac{a^4}{r^4}P_2=\frac{8}{3}eP_2^2+\ldots=\frac{8}{3}e.\frac{18}{35}P_4+\ldots=\frac{48}{35}eP_4+\ldots$

$\frac{a^6}{r^6}P_4=P_4$

$\frac{r}{a}(1-P_2)=\frac{2}{3}eP_2^2+\ldots=\frac{2}{3}e.\frac{18}{35}P_4+\ldots=\frac{12}{35}eP_4+\ldots$

It follows that the term in P_4 in $-dV/dr$ has a coefficient

$$\frac{M}{a^2}[\tfrac{16}{35}f-2(e-\tfrac{1}{2}m)\tfrac{48}{35}e+\tfrac{8}{7}(\tfrac{7}{2}e^2-\tfrac{5}{2}me-f)-\tfrac{2}{3}m.\tfrac{12}{35}e].$$

The term is therefore

$$-\tfrac{8}{35}\frac{M}{a^2}(3f-\tfrac{11}{2}e^2+\tfrac{15}{2}me)P_4.$$

Then adding the transverse component, it appears that the whole term is

$$-\tfrac{8}{35}\frac{M}{a^2}(3f-\tfrac{7}{2}e^2+\tfrac{15}{2}me)P_4.$$

But the coefficient of P_4 was shown to be $-\frac{8}{35}g_e a$; and since $g_e=M/a^2$ to the order zero, the coefficient in the expression for g is given by

$$\alpha=3f-\tfrac{7}{2}e^2+\tfrac{15}{2}me.$$

In the expression for gravity the geocentric colatitude θ is used, and it remains to introduce the true colatitude λ, which is connected with θ by the formulæ

$$\theta=\lambda+2e\sin\lambda\cos\lambda,$$

and
$$\cos^2\theta=\cos^2\lambda-4e\sin^2\lambda\cos^2\lambda.$$

Thus finally

$$\left.\begin{aligned} g&=g_e[1+\mathfrak{b}\cos^2\lambda-(\tfrac{5}{2}me-\tfrac{1}{2}e^2-3f)\sin^2\lambda\cos^2\lambda]\\ \text{where}\quad g_e&=\frac{M}{a^2}[1+e-\tfrac{3}{2}m+e^2-\tfrac{27}{14}me+\tfrac{9}{4}m^2-\tfrac{4}{7}f]\\ \mathfrak{b}&=\tfrac{5}{2}m-e-\tfrac{17}{14}me-\tfrac{2}{7}f. \end{aligned}\right\} \quad . \quad (40)$$

I shall show in § 9 that f is probably about $-\cdot00000205$; m is $\frac{1}{288\cdot41}$, or $\cdot0034672$, and if e be taken as $\frac{1}{298}$

$$\tfrac{5}{2}me-\tfrac{1}{2}e^2-3f=\cdot0000295.$$

Thus
$$g=g_e\{1+\mathfrak{b}\cos^2\lambda-\cdot0000295\sin^2\lambda\cos^2\lambda\} \quad . \quad (41)$$

The results of the pendulum experiments which have been made up to the present time are hardly sufficiently numerous or concordant amongst themselves to make it worth while to take into account this small term. If it represented a detectable inequality in gravity the residuals given on page 240 of volume ii. of Helmert's *Geodesy* would vary more or less as the square of the sine of twice the latitude; but they are quite irregular, being as follows:—

Latitude.	Residuals.	Latitude.	Residuals.
°		°	
5	−·0000139	45	−·0000015
15	− 054	55	− 73
25	+ 246	65	− 62
35	+ 008	75	+ 40

A great mass of material now awaits reduction, and we shall no doubt soon have from Dr. Helmert far more accurate results for gravity and for the ellipticity of the Earth's figure than any that have been obtained up to the present time.

§ 7. *The superficial values of the rates of increase of h and f.*

The rates of increase of h and f are given in (27) and (21); whence

$$a\frac{dh}{da} - 3h - \tfrac{27}{7}h^2 + \tfrac{76}{21}h\left(a\frac{dh}{da}\right) + \tfrac{11}{21}\left(a\frac{dh}{da}\right)^2 - \frac{\omega^2 a^3}{2\pi S_0}\left(a\frac{dh}{da}\right) + \frac{S_2}{S_0 a^2} = 0$$

$$a\frac{df}{da} - 5f + \tfrac{27}{2}h^2 - 7h\left(a\frac{dh}{da}\right) + \left(a\frac{dh}{da}\right)^2 - \tfrac{27}{2}\frac{S_4}{S_0 a^4} = 0.$$

But $$S_0 = \frac{3}{4\pi}M,$$

$$S_2 = \frac{5Ma^2}{4\pi}\left[h + \tfrac{2}{7}h^2 - \tfrac{1}{2}\frac{\omega^2 a^3}{M}(1 - \tfrac{2}{7}h)\right],$$

$$S_4 = \frac{Ma^4}{2\pi}\left[\tfrac{7}{2}h^2 - f - \tfrac{5}{2}\frac{\omega^2 a^3}{M}h\right].$$

Whence

$$a\frac{dh}{da} + 2h + \tfrac{18}{7}h^2 + \tfrac{76}{21}h\left(a\frac{dh}{da}\right) + \tfrac{11}{21}\left(a\frac{dh}{da}\right)^2 - \tfrac{5}{2}\frac{\omega^2 a^3}{M}\left(1 - \tfrac{2}{7}h + \tfrac{4}{15}a\frac{dh}{da}\right) = 0$$

$$a\frac{df}{da} + 4f - 18h^2 - 7h\left(a\frac{dh}{da}\right) + \left(a\frac{dh}{da}\right)^2 + \tfrac{45}{2}\frac{\omega^2 a^3}{M}h = 0.$$

But $\frac{\omega^2 a^3}{M} = m(1 - h - \frac{3}{2}m)$, so that the last term of the first of these equations may be written $-\frac{5}{2}m(1 - \frac{2}{7}h - \frac{3}{2}m + \frac{4}{15}a\frac{dh}{da})$, and the last term of the second is $+\frac{45}{2}mh$. The first equation then shows that to the first order $a\frac{dh}{da} = \frac{5}{2}m - 2h$. If this be used to eliminate dh/da from the terms of the second order in the first equation, and completely from the second, we get

$$\left.\begin{aligned} a\frac{dh}{da} &= \tfrac{5}{2}m - 2h + \tfrac{18}{7}h^2 - \tfrac{41}{7}mh - \tfrac{75}{14}m^2 \\ a\frac{df}{da} &= -4f + 5mh - \tfrac{25}{4}m^2. \end{aligned}\right\} \quad \ldots \quad (42)$$

The second of these may be combined with the first in the simple form

$$\frac{ad}{da}(f + \tfrac{5}{2}mh) + 4f = 0.$$

The parameter h denotes $e-\frac{2}{14}e^2-\frac{1}{7}f$; whence it may be shown that

$$\frac{a\,de}{da}=\tfrac{3}{2}\mathrm{m}-2\mathrm{e}-\mathrm{e}^2+\tfrac{33}{14}\mathrm{me}-\tfrac{25}{4}\mathrm{m}^2-\tfrac{2}{7}\mathrm{f}.$$

§ 8. *The Figure of a Homogeneous Mass with a small Nucleus.*

In order to obtain a preliminary estimate of the magnitude of f in the case of the Earth I shall first consider the case of a homogeneous mass with a nucleus at the centre of finite mass, but of infinitely small linear dimensions.

If we suppose the mass of the nucleus to be μ times that of the fluid, and that the form of surface is defined by the parameters a, h, f, it is clear from (10) and (11) that the external potential of the whole is

$$V=\tfrac{4}{3}\pi w\frac{a^3}{r}(1+\mu)-\tfrac{8}{15}\pi w\frac{a^5}{r^3}(h+\tfrac{2}{7}h^2)P_2+\tfrac{16}{35}\pi w\frac{a^7}{r^5}(h^2-\tfrac{2}{9}f)P_4$$
$$+\tfrac{1}{3}\omega^2r^2(1-P_2).$$

The equation to the surface will be of the form (8), and the transformations (12) hold good.

The conditions that the surface shall be level are, as before, that in V the coefficients of P_2 and P_4 shall vanish when r has the value (8).

These conditions are

$$\tfrac{2}{3}(1+\mu)a^2h(1-h)(1+\tfrac{20}{7}h)-\tfrac{2}{5}a^2h(1+\tfrac{2}{7}h)(1+\tfrac{11}{7}h)$$
$$-\frac{\omega^2a^2}{4\pi w}(1+\tfrac{2}{7}h)=0$$

$$8(1+\mu)a^2(f-\tfrac{1}{2}h^2)-\tfrac{72}{5}a^2h^2+\tfrac{8}{5}a^2(\tfrac{9}{2}h^2-f)+\frac{6\omega^2a^2}{\pi w}h=0\ ;$$

whence
$$h(1+\tfrac{11}{7}h)=\tfrac{15}{16}\frac{\omega^2}{\pi w}\cdot\frac{1}{1+\frac{5}{2}\mu}\quad\ldots\quad(43)$$

$$f(1+\tfrac{3}{2}\mu)=(\tfrac{6}{5}+\tfrac{3}{4}\mu)h^2-\frac{9\omega^2h}{8\pi w}.$$

To the first order of small quantities $\frac{\omega^2}{\pi w}=\frac{16}{15}(1+\frac{5}{2}\mu)h$, so that the second equation becomes

$$f=\frac{-\frac{3}{4}\mu h^2}{1+\frac{3}{2}\mu}\quad\ldots\quad(43)$$

If μ vanishes f also vanishes, and the surface is a true ellipsoid, as obviously should be the case.

Let us apply these formulæ (43) to the case of the Earth. It is known that ω^2a^3/M is about $\frac{1}{289}$, and it may be denoted by m, although the meaning of that symbol is slightly changed from that which it bears elsewhere.

Then $$m=\frac{\omega^2}{\frac{4}{3}\pi w(1+\mu)},$$ and

$$h(1+\tfrac{1}{7}h)=\tfrac{5}{4}m\frac{1+\mu}{1+\frac{5}{2}\mu}$$

$$f=-(\tfrac{5}{4}m)^2\frac{\frac{9}{4}\mu(1+\mu)^2}{(1+\frac{3}{2}\mu)(1+\frac{3}{2}\mu)^2}.$$

The equation which determines the value of μ which makes f a maximum is

$$\frac{1}{\mu}-\frac{3}{2+\mu}+\frac{2}{1+\mu}-\frac{10}{2+5\mu}=0.$$

This reduces to the quadratic $\mu^2-\frac{1}{4}\mu-\frac{1}{2}=0$, of which the positive root is

$$\mu=\tfrac{1}{8}[1+\sqrt{33}]=\cdot 84307.$$

The departure from the true ellipsoidal figure is greatest when the nucleus forms $\frac{843}{1843}$ or ·457 of the whole mass, and in this case

$$f=-\cdot 2946(\tfrac{5}{4}m)^2.$$

Taking $\frac{5}{4}m=\frac{1}{231}$,

$$f=-\cdot 0000055.$$

The Earth's equatorial radius is about 6,378,000 metres, and the depression of the surface below the true ellipsoid in latitude 45° is $\frac{1}{4}af$, which gives 8·8 metres for this ideal case.

It is clear that the depression in actuality must be considerably less than this, and we shall hereafter see reason to believe that it is about one-third of this maximum value.

§ 9. *Evaluation of the actual departure of the Earth's Figure from true Ellipticity.*

In order to determine the superficial value of f it is necessary to make some definite hypothesis as to the law of internal density.

The theory of the Earth's figure has been worked out according to several hypotheses as to the internal density. Two of these may be described as more prominent than others; they are the hypotheses of Laplace and of Roche. The results derived from both these theories are conformable to our knowledge, and as Roche's hypothesis seems more tractable than the other I shall adopt it.

Roche then supposes that the mean density w_0 of all the matter lying inside of equatorial radius a is expressed by a formula

$$w_0=\rho[1-k(\frac{a}{a})^2.]$$

The mean density of the whole Earth is clearly given by $w_0=\rho(1-k)$.

Since $w=w_0-\frac{1}{3}a\frac{dw_0}{da}$, it follows that

$$w=\rho[1-\tfrac{3}{3}k(\frac{a}{a})^2] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

This is only true as a first approximation, but it suffices for the present. I now put

$$x=k(\frac{a}{a})^2$$

and change the independent variable from a to x in the differential equations for f and h, the latter only being taken to the first order of small quantities.

The equations (31) and (23) then become

$$\left.\begin{aligned} &x(1-x)\frac{d^2h}{dx^2}+\tfrac{1}{2}(7-11x)\frac{dh}{dx}-h=0.\\ &x(1-x)\frac{d^2f}{dx^2}+\tfrac{7}{2}(1-x)(\frac{df}{dx}-\frac{f}{x})-2x(\frac{df}{dx}+\tfrac{1}{2}\frac{f}{x})+Q=0,\\ &\text{where}\\ &Q=2h^2+13h(x\frac{dh}{dx})+16(x\frac{dh}{dx})^2-\frac{1}{x}[7(x\frac{dh}{dx})+10(x\frac{dh}{dx})^2]. \end{aligned}\right\} \quad (45)$$

The first of these is Roche's equation, and I proceed to find the solution.*

If it be assumed that

$$h=E\sum_0^\infty H_n x^n,$$

where $H_0=1$, it is easy to prove that

$$H_{n+1}=\frac{n^2+\frac{9}{2}n+1}{n^2+\frac{9}{2}n+\frac{7}{2}}H_n.$$

Then if (n) denotes $\dfrac{n^2+\frac{9}{2}n+1}{n^2+\frac{9}{2}n+\frac{7}{2}}$

$$h=E[1+(0)x+(0)(1)x^2+(0)(1)(2)x^3+\ . \ .\ .].$$

* Tisserand, *Mécan. Cél.* vol. ii. A full account of the researches of Legendre, Laplace, Roche, Radau, Tisserand, Callandreau, and of others is given in this work.

Translating this into numbers I find

$$\frac{h}{E}=1+\frac{2}{7}x+\frac{13}{3^2.7}x^2+\frac{52}{3^3.11}x^3+\frac{47}{3^3.11}x^4+\frac{658}{3^4.5.11}x^5+\frac{31913}{3^5.5.11.17}x^6 + \ldots \quad (46)$$

From this I make the following series of deductions :—

$$\frac{h^2}{E^2}=1+\frac{4}{7}x+\frac{218}{3^2.7^2}x^2+\frac{6812}{3^3.7^2.11}x^3+\frac{20045}{3^4.7^2.11}x^4+\frac{3896}{3^5.5.7}x^5 + \frac{24210052}{3^6.5.7.11^2.17}x^6 \ldots \quad (47)$$

$$\frac{h}{E^2}\left(x\frac{dh}{dx}\right)=\frac{2}{7}x+\frac{218}{3^2.7^2}x^2+\frac{3406}{3^2.7^2.11}x^3+\frac{40090}{3^4.7^2.11}x^4+\frac{1948}{3^5.7}x^5 + \frac{24210052}{3^5.5.7.11^2.17}x^6 \ldots \quad (48)$$

$$\frac{1}{E}\left(x\frac{dh}{dx}\right)=\frac{2}{7}x+\frac{26}{3^2.7}x^2+\frac{52}{3^2.11}x^3+\frac{188}{3^3.11}x^4+\frac{658}{3^4.11}x^5+\frac{63826}{3^4.5.11.17}x^6 + \ldots \quad (49)$$

$$\frac{1}{E^2}\left(x\frac{dh}{dx}\right)^2=\frac{4}{7^2}x^2+\frac{104}{3^2.7^2}x^3+\frac{20540}{3^4.7^2.11}x^4+\frac{4960}{3^4.7.11}x^5+\frac{251176}{3^5.7.11^2}x^6+\ldots \quad (50)$$

By means of these developments I find that the function Q in (45) is given by

$$\frac{Q}{E^2}=\frac{256}{3^2.7^2}x+\frac{7024}{3^2.7^2.11}x^2+\frac{13120}{3^2.7^2.11}x^3+\frac{578128}{3^3.7^2.11}x^4+\frac{116120320}{3^4.5.7.11^2.17}x^5$$

Then, assuming $f=E^2\sum_1^\infty K_n x^n$, and substituting in the second of (45), we get

$$\Sigma[n(n+\tfrac{9}{2})K_{n+1}-(n^2+\tfrac{9}{2}n-\tfrac{5}{2})K_n]x^n+Q=0,$$

or $(\frac{11}{2}K_2-3K_1)x+(13K_3-\frac{21}{2}K_2)x^2+(\frac{45}{2}K_4-20K_3)x^3$
$+(34K_5-\frac{63}{2}K_4)x^4+(\frac{95}{2}K_6-45K_5)x^5+\ldots+Q=0$

The coefficients of the several powers of x are to be equated to zero, and the successive equations solved. Carrying out this process I find

$$\left.\begin{aligned}
K_2&=\frac{6}{11}K_1-\frac{512}{3^2.7^2.11} &&=\cdot54545K_1-\cdot15545\\
K_3&=\frac{63}{11.13}K_1-\frac{12400}{3^2.7^2.11.13} &&=\cdot44056K_1-\cdot19663\\
K_4&=\frac{56}{11.13}K_1-\frac{167424}{3^4.7^2.11.13} &&=\cdot39161K_1-\cdot29499\\
K_5&=\frac{882}{11.13.17}K_1-\frac{11668616}{3^5.7^2.11.13.17} &&=\cdot36281K_1-\cdot40312\\
K_6&=\frac{15876}{11.13.17.19}K_1-\frac{15778709488}{3^5.5.7^2.11^2.13.17.19} &&=\cdot34372K_1-\cdot52164
\end{aligned}\right\} \quad (51)$$

Extrapolated K_7 $=\cdot330K_1-\cdot635$

The coefficient K_1 remains indeterminate as yet, and I must now show how it is to be found.

The second of equations (20), (21), and (23) are as follows :—

$$f-\tfrac{3}{2}h^2+ah\frac{dh}{da}+\frac{3}{2w_0a^7}S_4-\frac{a^2}{3w_0}T_4=0 \quad \ldots\ldots\ldots\ldots (a)$$

$$\frac{df}{da}-\frac{5f}{a}+\frac{27}{2}\frac{h^2}{a}-7h\frac{dh}{da}+a\left(\frac{dh}{da}\right)^2-\frac{27}{2w_0a^8}S_4=0 \quad \ldots\ldots\ldots (b)$$

$$\frac{d^2f}{da^2}+6\frac{w}{w_0a}\frac{df}{da}-(20-6\frac{w}{w})\frac{f}{a^2}+12(1-\frac{w}{w_0})\frac{h^2}{a^2}+(4-18\frac{w}{w})\frac{hdh}{ada}$$
$$-(1+9\frac{w}{w})(\frac{dh}{da})^2=0 \ldots\ldots (c)$$

In obtaining (b) from (a) we multiplied (a) by w_0/a^2, differentiated, and divided by w_0/a^2. When x is independent variable we multiply by $(1-x)/x$, perform on the result the operation $x^3\frac{d}{dx}$, and divide by $(1-x)/x$.

Supposing all the functions to be expressed in series of powers of x, the equation (a) may be taken to be

$$P_1x+P_2x^2+\ldots+P_nx^n+\ldots=0 \quad \ldots\ldots (a')$$

It must be understood that the P's are numbers and not spherical harmonics. Then the equation (b) would be

$$(P_2-P_1)x^2+(2P_3-P_2-P_1)x^3+(3P_4-P_3-P_2-P_1)x^4$$
$$+(4P_5-P_4-P_3-P_2-P_1)x^5+\ldots=0 \quad (b')$$

Again, (c) was derived from (b) by multiplication by w_0a^8, differentiation, and division by w_0a^8. When x is independent variable we multiply by $x^4(1-x)$, perform the operation $x^5\frac{d}{dx}$, and divide by $x^4(1-x)$. Hence (c) must be equivalent to

$$11(P_2-P_1)x+(2.13P_3-15P_2-11P_1)x^2+(3.15P_4-19P_3-15P_2$$
$$-11P_1)x^3+(4.17P_5-21P_4-19P_3-15P_2-11P_1)x^4+\ldots=0 \quad (c')$$

Now the coefficients in f were evaluated by equating to zero the coefficients of the successive powers of x in (c) or (c'). Therefore

$$P_2-P_1=0,\ 2.13P_3-15P_2-11P_1=0, \text{ and so forth.}$$

These equations are satisfied by

$$P_1=P_2=P_3=\ldots=P_n;$$

but one of the coefficients, say P_1, remains indeterminate.

The equation (b') is satisfied by the same condition ; but in order that (a') may be satisfied it is necessary not only that all the P's should be equal, but that they should also vanish.

Accordingly (a) affords us one more condition from which K_1 will be determinable.

When x is the independent variable, the equation (a) may be written

$$f-\tfrac{3}{2}h^2+2h\left(\frac{x\,dh}{dx}\right)+\tfrac{3}{2}x^{\frac{1}{2}}\frac{1}{(1-x)}\int_0^x(1-\tfrac{5}{3}x)\,d[x^{\frac{1}{2}}(h^2-\tfrac{2}{3}f)]$$

$$-\frac{x}{3(1-x)}\int_x^k(1-\tfrac{5}{3}x)\,d\left(\frac{f}{x}\right)=0.$$

Since f/E^2 is expressible by a series beginning with K_1x, we shall be able to find K_1 by developing this equation in a series, but only carrying the development out as far as the first term.

I drop the factor E^2 for the sake of brevity.

Then, $f=K_1x$; $h^2=1+\frac{4}{7}x$, from (47) ; $h^2-\frac{2}{3}f=1+\frac{4}{7}x-\frac{2}{3}K_1x$;

$$\frac{d}{dx}[x^{\frac{1}{2}}(h^2-\tfrac{2}{3}f)]=x^{\frac{1}{2}}(\tfrac{7}{2}+\tfrac{18}{7}x-K_1x),$$

$$(1-\tfrac{5}{3}x)\frac{d}{dx}[x^{\frac{1}{2}}(h^2-\tfrac{2}{3}f]=x^{\frac{1}{2}}(\tfrac{7}{2}-\tfrac{137}{42}x-K_1x)\,;$$

$$\int_0^x(1-\tfrac{5}{3}x)\,d[x^{\frac{1}{2}}(h^2-\tfrac{2}{3}f)]=x^{\frac{1}{2}}(1-\tfrac{137}{189}x-\tfrac{2}{9}K_1x)\,;$$

whence

$$2x^{\frac{1}{2}}\frac{3}{(1-x)}\int_0^x(1-\tfrac{5}{3}x)\,d[x^{\frac{1}{2}}(h^2-\tfrac{2}{3}f)]=\tfrac{3}{2}+\tfrac{26}{63}x-\tfrac{1}{3}K_1x.$$

Also
$$-\tfrac{3}{2}h^2+2h\left(\frac{x\,dh}{dx}\right)=-\tfrac{3}{2}-\tfrac{2}{7}x.$$

Therefore as far as the first power of x the equation is

$$K_1x+\tfrac{8}{63}x-\tfrac{1}{3}K_1x-\tfrac{1}{3}x\int_0^k(1-\tfrac{5}{3}x)\,d\left(\frac{f}{x}\right)=0.$$

Thus, reintroducing the factor E^2,

$$K_1=-\tfrac{4}{21}+\frac{1}{2E^2}\int_0^k(1-\tfrac{5}{3}x)\,d\left(\frac{f}{x}\right).$$

But $\dfrac{f}{E^2x}=K_1+K_2x+\ldots+K_nx^{n-1}+\ldots,$

Therefore

$$(1-\tfrac{5}{3}x)\frac{d}{dx}\left(\frac{f}{E^2x}\right)=K_2+(2K_3-\tfrac{5}{3}K_2)x+\ldots$$

$$+[(n+1)K_{n+2}-\tfrac{5}{3}nK_{n+1}]x^n+\ldots$$

and

$$\frac{1}{E^2}\int_0^k (1-\tfrac{5}{3}x)d\left(\frac{f}{x}\right) = K_2 k + (K_3 - \tfrac{5}{6}K_2)k^2 + \ldots + (K_{n+2} - \frac{5n}{3(n+1)}K_{n+1})k^{n+1} + \ldots$$

On substituting for the K's their values, I find

$$K_1 + \tfrac{4}{21} = K_1(\cdot 27273k - \cdot 00699k^2 - \cdot 04895k^3 - \cdot 06335k^4 - \cdot 07002k^5 - \cdot 0737k^6 \ldots) - (\cdot 07773k + \cdot 03355k^2 + \cdot 03826k^3 + \cdot 01719k^4 - \cdot 00793k^5 - \cdot 04475k^6 \ldots,)$$

To find k we have the equations

$$2\frac{\Sigma n A_n k^n}{\Sigma A_n k^n} = 2\frac{k dh}{h dk} = \frac{a dh}{h da}$$

$$= \tfrac{3}{2}\frac{m}{h} - 2 + \tfrac{18}{7}h - \tfrac{4}{7}m - \tfrac{11}{14}\frac{m^2}{h}.$$

$$h = e - \tfrac{9}{14}e^2 - \tfrac{1}{7}f.$$

The solution is virtually contained in the table of § 12 below.* From this it appears that when e is $\frac{1}{20\cdot 7}$, k is ·464.

Now with k=·464 I find

$$K_1(\cdot 11426) - \cdot 0464 = K_1 + \frac{4}{21};$$

whence on substitution in (51)

$$K_1 = -\cdot 2674$$
$$K_2 = -\cdot 3013$$
$$K_3 = -\cdot 3144$$
$$K_4 = -\cdot 3997$$
$$K_5 = -\cdot 5002$$
$$K_6 = -\cdot 6136$$

and $K_7 = -\cdot 72$, by extrapolation.

With these values for the K's, and with k=·464,

$$f = -\cdot 2632E^2.$$

But with k=·46, $\frac{h}{E} = 1\cdot 20465$, and with k=·47, $\frac{h}{E} = 1\cdot 21193$; whence with k= −·464, $E = \frac{1}{358\cdot 5}$.

* In making the computations I treated f as zero.

Hence finally

$$\left.\begin{aligned} f &= -\cdot 00002 05 \\ \tfrac{1}{4}af &= -3\cdot 26 \text{ metres} \end{aligned}\right\} \quad \ldots\ldots\ldots (52)$$

This result shows that the Earth's surface is $3\frac{1}{4}$ metres below the ellipsoid in latitude 45°.

I have already used this value of f in the evaluation of gravity in equation (41) § 7.

M. Callandreau has not solved his differential equation which corresponds with mine, but he concludes that the depression in latitude 45° must be less than 5 metres.*

§ 10. *The departure from true Ellipticity according to Professor Wiechert's hypothesis.*

Professor Wiechert† has adduced forcible arguments in favour of the hypothesis that the Earth consists of an iron nucleus, of approximately uniform density, with a superposed layer of rock. He concludes that the nucleus must occupy about four-fifths of the radius. He considers the conditions that both nucleus and surface may be level, and gives valuable numerical tables.

The method of this paper permits us to give the conditions from which his tables were computed somewhat more succinctly than he does. I will therefore give my results in outline.

Let ρ', ρ be the densities of the nucleus and of the superficial layer; let a', e' or h', f' define the figure of the nucleus, and a, e or h, f that of the surface; also let ρ_0 be the mean density of the whole.

It is clear that

$$\left.\begin{aligned} \rho_0 &= \rho + (\rho' - \rho)\left(\frac{a'}{a}\right)^3 (1 + h - h'), \\ \text{whence } \frac{\rho'}{\rho_0} &= \frac{\rho}{\rho_0} - \left(1 - \frac{\rho}{\rho_0}\right)\left(\frac{a}{a}\right)^3 (1 - h + h') \end{aligned}\right\} \quad \ldots (53)$$

The mean density ρ_0 may be taken as known, but if the values of ρ and of a'/a be assumed, the value of ρ' can only be rigorously found after the determination of h, h'. As a first approximation, very near the truth however, we may take $h = h'$, so that

$$\frac{\rho'}{\rho_0} = \frac{\rho}{\rho_0} - \left(1 - \frac{\rho}{\rho_0}\right)\left(\frac{a}{a'}\right)^3.$$

The integrals S_0, S_2, S_4, T_2, T_4, defined in (10) are only

* 'Ann. de l'Obs. de Paris,' *Mémoires*, t. xix. 1889, p. E. 51.

† 'Ueber die Massenvertheilung im Innern der Erde,' *Nachr. K. Gesell. zu Göttingen*, 1896–7, p. 221.

required when $a=a'$ at the boundary of the nucleus, and when $a=a$ at the surface.

When $a=a'$

$$S_0=\rho' a'^3(1-h')\ ;\ S_2=\rho' a'^5 h'(1+\tfrac{2}{7}h')\ ;\ S_4=\rho' a'^7(h'^2-\tfrac{2}{9}f')\ ;$$

$$T_2=\rho(h-h')(1+\tfrac{1}{7}^7 h+\tfrac{1}{7}^7 h')\ ;\qquad T_4=\rho(\frac{f}{a^2}-\frac{f'}{a'^2}).$$

Similarly, the integrals corresponding to $a=a$ may be written down. If ρ' be then eliminated by means of (53), it will be found that they are

$$S_0=\rho_0 a^3(1-h)$$

$$S_2=\rho_0 a^3[(1-\frac{\rho}{\rho_0})h'(1-h+\tfrac{2}{7}h')a'^2+\frac{\rho}{\rho_0}h(1+\tfrac{2}{7}h)a^2]$$

$$S_4=\rho_0 a'^3[(1-\frac{\rho}{\rho_0})(h'^2-\tfrac{2}{9}f')a'^4+\frac{\rho}{\rho_0}(h^2-\tfrac{2}{9}f)a^4]$$

$$T_2=0,\qquad T_4=0.$$

The conditions that the surfaces a' and a may be level surfaces are given in (13) and (16), with the above values for the S and T integrals. I find that a considerable simplification in the expression results from considering $h(1+\frac{1}{7}^1 h)$, $h'(1+\frac{1}{7}^1 h')$ as the unknown quantities instead of simply h and h'. Accordingly I write

$$k=h(1+\tfrac{1}{7}^1 h),\qquad k'=h'(1+\tfrac{1}{7}^1 h').$$

It may be remarked in passing that $k=e-\frac{3}{14}e^2-\frac{1}{7}f$.

Without giving details I may state that the condition (13) leads to the two equations—

$$\left.\begin{aligned} k'(\frac{\rho'}{\rho_0}+\tfrac{3}{2}\frac{\rho}{\rho_0})-k(\tfrac{3}{2}\frac{\rho}{\rho_0})&=\tfrac{1}{1}\tfrac{5}{6}\frac{\omega^2}{\pi\rho_0}+\tfrac{1}{7}\frac{\rho}{\rho_0}(k-k')(9k-5k')\\ -\tfrac{3}{2}k'(\frac{a'}{a})^2(1-\frac{\rho}{\rho_0})+k(\tfrac{5}{2}-\tfrac{3}{2}\frac{\rho}{\rho_0})&=\tfrac{1}{1}\tfrac{5}{6}\frac{\omega^2}{\pi\rho_0}+\tfrac{3}{7}(1-\frac{\rho}{\rho_0})k'(k-k')(\frac{a'}{a})^2 \end{aligned}\right\}\quad(54)$$

We might first neglect the small terms on the right, and use the approximate value of ρ' on the left, solve the equations and then proceed to a second approximation. The approximate solution

$$\frac{k}{(2\rho'+3\rho)a^2+3(\rho_0-\rho)a'^2}=\frac{k'}{5\rho_0 a^2}$$

$$=\frac{15\omega^2}{8\pi\rho_0}\Big/(5\rho_0-3\rho)(2\rho'+3\rho)a^2-9\rho(\rho_0-\rho)a'^2$$

is, however, very near the truth, because the small terms on the

right not only depend on the squares of the ellipticity, but also involve $k-k'$, which is itself much smaller than either k or k'.

In these equations we have, of course, $\frac{15\omega^2}{8\pi\rho_0}=\frac{5}{2}m(1-\frac{3}{2}m)$.

Now turning to the condition (16), in which it is clearly permissible to write k, k', for h, h', I find

$$f(\frac{2\rho'}{\rho}+1)-f'(\frac{a'}{a})^2=0,$$

$$-f(1-\tfrac{1}{3}\frac{\rho}{\rho_0})+\tfrac{1}{3}f'(1-\frac{\rho}{\rho_0})(\frac{a'}{a})^4=\tfrac{3}{2}(1-\frac{\rho}{\rho_0})\{k-k'(\frac{a'}{a})^2\}.$$

From this

$$f=\frac{-\frac{3}{2}a^2(ka^2-k'a'^2)^2}{(3+2\frac{\rho}{\rho_0-\rho})a^6-\frac{a'^6}{1+2\rho'/\rho}} \quad . \quad . \quad . \quad . \quad (55)$$

If ρ' were infinite and a' zero, but $\rho'a'^3=\mu\rho a^3(1-h)$, we ought to come back on the result for the nucleus of infinitely small dimensions in § 8; and this is so.

In one of the cases considered by Professor Wiechert he took $\rho=3{\cdot}2$, $\rho'=8{\cdot}206$, $a'/a={\cdot}78039$, and found the ellipticity of the surface to be $\frac{1}{297}$. The case corresponds closely with that of the Earth. He also computed a certain function from which my f might be derived. I find, however, by computation directly from (55), that $f=-{\cdot}00000175$, and $\frac{1}{4}af=-2{\cdot}79$ metres.

With Roche's hypothesis I found $f=-{\cdot}00000205$, and $\frac{1}{4}af=-3{\cdot}26$ metres. Thus the two hypotheses lead to nearly the same result.

§ 11. *Solution of the Differential Equation for h.*

I propose to solve the equation (31) with Roche's hypothesis, and thus to estimate the contribution of the terms of the second order to the value of h.

As in § 9, x is to be the independent variable.

The expression for w in (44) is no longer sufficient, but a term of the first order must be included in it.

It was proved in (32) that

$$w=w_0[1+\tfrac{1}{3}(1+\tfrac{1}{3}a\frac{dh}{da}\frac{adw_0}{w_0da}].$$

Now $w_0=\rho(1-x)$ and $a\frac{d}{da}=2x\frac{d}{dx}$, so that

$$w=\rho[1-\tfrac{5}{3}x-\tfrac{4}{9}x(x\frac{dh}{dx})],$$

and

$$w_0-w=\tfrac{2}{3}\rho x[1+\tfrac{2}{3}(x\frac{dh}{dx})] \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

When x is introduced as independent variable in (31), and when the equation is divided by 4, the terms of the first order assume the form given in the first of (45). Then by aid of (55) the terms of the second order are easily determined, and the equation will be found to assume the following form :—

$$\left.\begin{aligned} &x(1-x)\frac{d^2h}{dx^2}+\tfrac{1}{2}(7-11x)\frac{dh}{dx}-h+P-R=0 \\ \text{where}\quad &P=\tfrac{7}{3}h^2+\tfrac{5}{21}[h(x\frac{dh}{dx})+(x\frac{dh}{dx})^2] \\ &R=\tfrac{2}{3}m(h+2x\frac{dh}{dx}). \end{aligned}\right\} \quad . \quad . \quad (56)$$

In the last of these m denotes $\frac{3\omega^2}{4\pi w_0}$; this is equal to $\frac{3\omega^2}{4\pi w_0}\cdot\frac{w_0}{w_0}$ or $m\frac{1-k}{1-x}$. Accordingly

$$R=\tfrac{2}{3}m(1-k)\frac{h+2x\dfrac{dh}{dx}}{1-x} \quad . \quad . \quad . \quad . \quad . \quad (56)$$

By means of the series (47), (48), and (49) I find

$$P=E^2\{\tfrac{7}{3}+\frac{100}{7^2}x+\frac{23890}{3^4.7^3}x^2+\frac{149081}{3^4.7^2.11}x^3+\frac{4134965}{3^5.7^3.11}x^4+\frac{11540024}{3^6.5.7^2.11}x^5 + \frac{1658730884}{3^7.7^2.11^2.17}x^6+\ldots\}$$

or

$$P=E^2\{\tfrac{7}{3}+2\cdot0408x+2\cdot5796x^2+3\cdot4149x^3+4\cdot5100x^4 +5\cdot8738x^5+7\cdot5248x^6+9\cdot45x^7+\ldots\}$$

the last term being extrapolated.

Now we saw in the last section that for the Earth the reciprocal of E was 358·5. As a fact I have here used the value 361·8, and it does not seem worth while to recompute on account of this small change in E.

Then

$$I'=E\{\cdot006450+\cdot005641x+\cdot007131x^2+\cdot009440x^3 +012467x^4+016237x^5+\cdot020801x^6+\cdot0261x^7+\ldots\}$$

When the fractions in (46) and (49) are expressed in decimals, I find

$$h+2x\frac{dh}{dx}=E\{1+\cdot85714x+1\cdot03175x^2+1\cdot22559x^3 +1\cdot42424x^4+1\cdot62469x^5+1\cdot82597x^6+2\cdot028x^7+\ldots\}$$

And

$$\frac{h+2x\dfrac{dh}{dx}}{1-x}=E\{1+1\cdot85714x+2\cdot88889x^2+4\cdot11448x^3 +5\cdot53872x^4+7\cdot16341^5+8\cdot98938x^6+11\cdot016x^7\ \ldots\}$$

This last must be multiplied by $\frac{2}{3}\mathrm{m}(1-\mathrm{k})$ in order to find R. Now m=·00346, k=·464 ; so that the factor is ·001236. Then

$$R=E\{\cdot001236+\cdot002296x+\cdot003572x^2+\cdot005087x^3 +\cdot006848x^4+\cdot008857x^5+\cdot011114x^6+\cdot0136x^7+\ \ldots\}$$

Finally

$$\frac{P-R}{E}=\cdot005214+\cdot003345x+\cdot003559x^2+\cdot004353x^3 +\cdot005619x^4+\cdot007380x^5+\cdot009687x^6+\cdot0125x^7+\ \ldots$$

I now assume as before,

$$h=E\sum_0^\infty H_n x^n,$$

so that the differential equation (56) gives

$$\Sigma[(n^2+\tfrac{3}{2}n+\tfrac{7}{2})H_{n+1}-(n^2+\tfrac{3}{2}n+1)H_n]x^n+\frac{P-R}{E}=0.$$

When the coefficients of the successive powers of x are equated to zero the first equation is

$$\tfrac{7}{2}H_1-H_0=-\cdot005214.$$

If we assume $H_0=1$, which is clearly permissible,

$$H_1=\tfrac{2}{7}-\cdot001490.$$

Then from the successive equations I find

$$H_2=\frac{13}{3^2.7}-\cdot001448$$

$$H_3=\frac{52}{3^3.11}-\cdot001444$$

$$H_4=\frac{47}{3^3.11}-\cdot001473$$

$$H_5=\frac{658}{3^4.5.11}-\cdot001524$$

$$H_6=\frac{31913}{3^5.5.11.17}-\cdot001594.$$

$$H_7=\frac{583552}{3^5.5.11.17.19}-001698.$$

The first part of each of these coefficients is the corresponding coefficient in the first approximation, given in (46). If, therefore, we denote by δh the correction to be applied to the first approximation as arising from the terms of the second order, we have

$$\delta h = -E\{\cdot00149x + \cdot00145x^2 + \cdot00144x^3 + \cdot00147x^4 + \cdot00152x^5 + \cdot00159x^6 + \cdot00170x^7 + \ldots\}$$

By an empirical summation of this series, we may assert that a fair approximation to the result is given by

$$\delta h = -\cdot0015\frac{Ex}{1-x}.$$

But $\frac{2}{3}\frac{x}{1-x} = 1 - \frac{w}{w_0}$, so that the correction becomes

$$\delta h = -\cdot00225E\left(1-\frac{w}{w_0}\right).$$

or since E is about $\frac{1}{360}$,

$$\delta h = -\cdot000006 2\left(1-\frac{w}{w_0}\right).$$

The superficial value of h corresponding to $e = \frac{1}{297}$ is ·003349, and

$$\frac{\delta h}{h} = -\cdot0019\left(1-\frac{w}{w_0}\right).$$

It is, however, better to look at this correction from another point of view. If h_0 denotes h as derived from the first approximation, we have, by means of the empirical solution,

$$h = h_0 - \cdot0015\frac{Ex}{1-x};$$

so that

$$x\frac{dh}{dx} = x\frac{dh_0}{dx} - \cdot0015\frac{Ex}{(1-x)^2}.$$

We have seen in § 9 that k in Roche's hypothesis is derived from the equation

$$\frac{2\Sigma nA_n k^n}{\Sigma A_n k^n} = \frac{5}{2}\frac{m}{h} - 2 + \frac{18}{7}h - \frac{41}{7}m - \frac{75}{14}\frac{m^2}{h}.$$

It now appears that, when terms of the second order are included in the differential equation for h, the left-hand side of this equation becomes

$$\frac{2\left(\Sigma nA_n k^n - \cdot0015\frac{k}{(1-k)^2}\right)}{\Sigma A_n k^n - \cdot0015\frac{k}{1-k}}$$

Now I found that when $k=\cdot464$, $\Sigma A_n k^n=1\cdot20756$, $\Sigma nA_n k^n=\cdot33666$.

Therefore

$$\Sigma nA_n k^n-\cdot0015\frac{k}{(1-k)^2}=\Sigma nA_n k^n\left(1-\frac{150}{33666}\cdot\frac{\cdot464}{(\cdot536)^2}\right)$$

$$=\Sigma nA_n k^n(1-\cdot00720).$$

$$\Sigma A_n k^n-\cdot0015\frac{k}{1-k}=\Sigma A_n k_n\left(1-\frac{150}{120756}\cdot\frac{464}{536}\right)$$

$$=\Sigma A_n k^n(1-\cdot00108).$$

Since $\frac{1-\cdot00720}{1-\cdot00108}=1-\cdot00612=\cdot99388$, it follows that the left-hand side of the equation for k is

$$\cdot994\left(\frac{2\Sigma nA_n k^n}{\Sigma A_n k^n}\right).$$

The principal effect of the terms of the second order will therefore be slightly to alter the value of k which satisfies the observed conditions. In every hypothesis as to the internal density there is some parameter derivable from observation, and a similar investigation would show that its value is but slightly affected by the terms of the second order. It is clear, then, that there is not much to be gained by pursuing this investigation further.

§ 12. *The Moments of Inertia, the Precessional Constant, and the Ellipticity of the Earth.*

It was shown in (6) of § 1 that

$$C=\tfrac{8}{15}\pi\int_0^a wd[a^5(1-e+\tfrac{4}{7}f)].$$

If the terms of the second order be omitted we may drop f and replace e by h. Also to the first order $S_2=\int_0^a wd(a^5h)$; and by (35)

$$S_2=\frac{5M}{4\pi}a^2[h-\tfrac{1}{2}m].$$

Hence $$C=\tfrac{8}{3}\pi\int_0^a wa^4da-\tfrac{2}{3}Ma^2(h-\tfrac{1}{2}m).$$

It was shown in (32) that

$$w=w_0+\tfrac{1}{3}a\frac{dw_0}{da}+\tfrac{1}{5}a^2\frac{dh}{da}\frac{dw_0}{da},$$

so that

$$\int_0^a wa^4da=\int_0^a w_0a^4da+\tfrac{1}{3}\int_0^a a^5dw_0+\tfrac{1}{9}\int_0^a a^6\frac{dh}{da}dw_0.$$

If the middle term in this expression be integrated by parts we have a term $-\frac{5}{3}\int_0^a w_0a^4da$, which will fuse with the first term. Therefore

$$\int_0^a wa^4da=\tfrac{1}{3}\mathrm{w_0a^5}-\tfrac{2}{3}\int_0^a w_0a^4da+\tfrac{1}{9}\int_0^a a^6\frac{dh}{da}dw_0,$$

and

$$C=\tfrac{8}{9}\pi\mathrm{w_0a^5}\left[1-\frac{2}{\mathrm{w_0a^5}}\int_0^a w_0a^4da+\frac{1}{3\mathrm{w_0a^5}}\int_0^a a^6\frac{dh}{da}dw_0\right]-\tfrac{2}{5}\mathrm{Ma^2(h-\tfrac{1}{2}m)}$$

But to the order of approximation here adopted

$$\tfrac{8}{9}\pi\mathrm{w_0a^5}=\tfrac{2}{3}\mathrm{Ma^2}(1+\mathrm{h}).$$

Therefore

$$C=\tfrac{2}{3}\mathrm{Ma^2}\left[1-\frac{2(1+\mathrm{h})}{\mathrm{w_0a^5}}\int_0^a w_0a^4da+\frac{1}{3\mathrm{w_0a^5}}\int_0^a a^6\frac{dh}{da}dw_0+\tfrac{1}{2}\mathrm{m}\right].$$

We now make use of M. Radau's transformation. It was proved in (34), § 5, that

$$\mathrm{w_0a^5}\surd(1+\eta_1)=5(1+\lambda)\int_0^a w_0a^4da,$$

where $1+\lambda$ is a mean value of a certain function.

Therefore

$$-\frac{2(1+\mathrm{h})}{\mathrm{w_0a^5}}\int_0^a w_0a^4da=-\tfrac{2}{5}\frac{(1+\mathrm{h})\surd(1+\eta_1)}{1+\lambda}.$$

Now

$$\int_0^a a^6\frac{dh}{da}dw_0=\mathrm{w_0a^6}\frac{\mathrm{dh}}{\mathrm{da}}-\int_0^a w_0(6a^5\frac{dh}{da}+a^6\frac{d^2h}{da^2})da.$$

But h satisfies the differential equation

$$\frac{d^2h}{da^2}+6\frac{w}{w_0}\frac{dh}{ada}-6(1-\frac{w}{w_0})\frac{h}{a^2}=0,$$

and

$$\frac{dw_0}{da}=-\frac{3}{a}(w_0-w).$$

Therefore

$$\int_0^a w_0(6a^5\frac{dh}{da}+a^6\frac{d^2h}{da^2})da=6\int_0^a(w_0-w)\,(ha^4+a^5\frac{dh}{da})da$$

$$=6\int_0^a(w_0-w)ha^4da-2\int_0^a a^6\frac{dh}{da}dw_0.$$

Therefore

$$\int_0^a a^6 \frac{dh}{da} dw_0 = -w_0 a^5 h \eta_1 + 6 \int_0^a (w_0 - w) h a^4 da.$$

Now let

$$\left.\begin{aligned} \Delta_1 &= \tfrac{2}{5}(1+h)\sqrt{(1+\eta_1)}\,\frac{\lambda}{1+\lambda} \\ \Delta_2 &= -\tfrac{1}{3} h \eta_1 + \frac{2}{w_0 a^5} \int_0^a (w_0 - w) h a^4 da \\ \Delta &= \Delta_1 + \Delta_2 \end{aligned}\right\} \quad . \quad . \quad . \quad (57)$$

and we have

$$C = \tfrac{2}{3} M a^2 \left[1 - \tfrac{2}{5}(1+h)\sqrt{(1+\eta_1)} + \tfrac{1}{2} m + \Delta\right] \quad . \quad . \quad (57)$$

The only part of the expression for C which involves the law of internal density is Δ, and we shall see that Δ is very small compared with unity.

M. Callandreau has established a formula for C with which, as far as I can make out, mine agrees.*

It is clear that the two parts of Δ originate in quite different ways, Δ_1 depending on the equation for the internal ellipticity, and Δ_2 depending on the terms of the second order.

It is first necessary to evaluate λ from the equation

$$1 + \lambda = \frac{\frac{1}{5} w_0 a^5 \sqrt{(1+\eta_1)}}{\int_0^a w_0 a^4 da} \quad . \quad . \quad . \quad . \quad . \quad (58)$$

where $$1 + \eta_1 = \tfrac{5}{2}\frac{m}{h} - 1 + \tfrac{18}{7} h - \tfrac{41}{7} m - \tfrac{75}{14}\frac{m^2}{h}.$$

It is not hard to obtain a close approximation to λ when Laplace's hypothesis as to the density is adopted, but that hypothesis is not so tractable as Roche's, and I therefore adopt the latter.

According to Roche

$$w_0 = \rho(1 - x), \text{ where } x = k\left(\frac{a}{a}\right)^2.$$

Therefore

$$\begin{aligned} \int_0^a w_0 a^4 da &= \tfrac{1}{2} \frac{\rho a^5}{k^{\frac{5}{2}}} \int_0^k (1-x) x^{\frac{3}{2}} dx \\ &= \tfrac{1}{5} \rho a^5 (1 - \tfrac{5}{7} k). \end{aligned}$$

Accordingly $$1 + \lambda = \frac{1-k}{1 - \frac{5}{7}k} \sqrt{(1+\eta_1)}.$$

Since $w_0 = \rho\,(1-k)$, and with close approximation,

* *Bulletin Astronomique*, vol. xiv. 1897, p. 217.

$w=\rho(1-\frac{5}{3}k)$, we have $\rho=(\frac{5}{2}w_0-\frac{3}{2}w)$, $k=\dfrac{3(w_0-w)}{5w_0-3w}$ very nearly.
Hence we may also write

$$1+\lambda=\frac{\sqrt{(1+\eta_1)}}{1+\frac{3}{7}(1-w/w_0)}\ *\quad .\quad .\quad .\quad .\quad .\quad (59)$$

Therefore

$$\Delta_1=\tfrac{2}{5}(1+h)[\sqrt{(1+\eta_1)}-1-\tfrac{3}{7}(1-w/w_0)]\quad .\quad .\quad (60)$$

It is now required to evaluate Δ_2.
With Roche's hypothesis

$$\frac{2}{w_0a^5}\int_0^a(w_0-w)ha^4da=\tfrac{2}{3}\frac{\rho}{w_0}\cdot\frac{1}{k^{\frac{5}{3}}}\int_0^k hx^{\frac{5}{2}}dx.$$

I write the series (46) $h=E\overset{\infty}{\underset{0}{\Sigma}}A_nx^n$, and then

$$\frac{2}{w_0a^5}\int_0^a(w_0-w)ha^4da=\tfrac{2}{3}\frac{E\rho}{w_0k^{\frac{5}{2}}}\Sigma\int_0^k A_nx^{n+\frac{5}{2}}dx,$$

$$=\tfrac{4}{3}\frac{Ek}{1-k}\Sigma\frac{1}{2n+7}A_nk^n.$$

But $h=E\Sigma A_nk^n$, and therefore

$$\Delta_2=\tfrac{1}{3}h\left[-\eta_1+\frac{4k\Sigma\frac{1}{2n+7}A_nk^n}{(1-k)\Sigma A_nk^n}\right]$$

or

$$\Delta_2=\tfrac{1}{3}h\left[-\eta_1+6(1-\frac{w}{w_0})\frac{\Sigma\frac{1}{2n+7}A_nk^n}{\Sigma A_nk^n}\right]\quad .\quad .\quad .\quad (61)$$

In order to estimate the magnitude of the correction Δ I have computed it, and its two constituents, in the following table :—

* The approximation here consists in neglecting the variation of ellipticity in evaluating the density w from the mean density w_0. In Laplace's hypothesis $w=\rho\frac{\kappa}{a}\sin\frac{a}{\kappa}$. If we neglect the variation of ellipticity in determining the mean density w_0 from the density w, it will be found that

$$1+\lambda=\frac{1}{15}\frac{a^2}{\kappa^2}\frac{\sqrt{(1+\eta_1)}}{1-w/w_0}.$$

Since $\dfrac{w_0}{w}=\dfrac{\kappa^2}{a^2}\left(1-\dfrac{a}{\kappa}\cot\dfrac{a}{\kappa}\right)$, κ may be regarded as a function of w_0/w, and λ is seen to be a function of w_0/w and of η_1, as in the text.

Table of Results according to Roche's Hypothesis.

$\left(m = \frac{1}{288{\cdot}41}\right)$.

	k	·1	·2	·3	·4	·46	·47	·5	·55
Ratio of mean to surface density	w_0/w	= 1·08	1·2	1·4	1·8	2·314	2·446	3·0	5·4
Reciprocal of ellipticity	$\frac{1}{e}$	= 239·3	248·2	262·2	280·7	295·7	298·6	308·2	327·5
	$h \times 10^6$	= 4148	4001	3788	3541	3362	3329	3227	3037
Moment of inertia, uncorrected	$C \div \frac{2}{3} Ma^2$	= ·58728	·57137	·55088	·52360	·50245	·49847	·48557	·46036
	$\lambda \times 10^6$	= 47	184	387	548	516	493	422	99
First correction	$\Delta_1 \times 10^6$	= 19	79	174	262	258	248	218	54
Second correction ...	$\Delta_2 \times 10^6$	= − 2	− 10	− 28	− 63	− 95	−102	−125	−173
Ratio of total correction to moment of inertia...	$\frac{\frac{2}{3}Ma^2\Delta}{C} \times 10^6$	= 29	120	265	380	322	293	192	−258

It will be observed that λ and Δ_1 reach maxima when k is ·4, whereas $-\Delta_2$ increases throughout as k increases. The last line gives the factor of augmentation of the uncorrected value of C. The columns k=·46 and ·47 are those which correspond to the case of the Earth most closely, and they show that C must be augmented by a factor 1·0003 when Δ is taken into account.

We are now in a position to give the formula for the Precessional Constant $\frac{C-A}{C}$.

It appears from (9), (10), (35), and (36) that

$$C-A=\tfrac{2}{3}Ma^2[h-\tfrac{1}{2}m+\tfrac{2}{7}h^2+\tfrac{1}{7}mh+\tfrac{3}{4}m^2] \quad . \quad . \quad (61)$$

So that, dividing (61) by (57),

$$\left.\begin{aligned}\frac{C-A}{C}&=\frac{h-\frac{1}{2}m+\frac{2}{7}h^2+\frac{1}{7}mh+\frac{3}{4}m^2}{1-\frac{2}{5}(1+h)\sqrt{(1+\eta_1)}+\frac{1}{2}m+\Delta'}\\ \text{where}\quad 1+\eta_1&=\tfrac{5}{2}\frac{m}{h}-1+\tfrac{18}{7}h-\tfrac{41}{7}m-\tfrac{755}{14}\frac{m^2}{h}\end{aligned}\right\} \quad . \quad (62)$$

We have reason to believe that the term Δ may be allowed for by first treating it as zero, and afterwards multiplying the result by ·9997.

The Precessional Constant is known with a high degree of accuracy, and I cannot but think that this investigation shows that it may be used for determining the actual ellipticity of the Earth's surface with perhaps as little error as by any other method. The uncertainty is, indeed, of a different kind, being dependent on our ignorance of the interior of the Earth. I reduce the formula (62) to numbers in the following manner. I assume a definite value for the ellipticity, namely $e_0=\frac{1}{299}$ =·00334448. Then, with f=−·00000205, h_0 is computed and found to be ·00332480. I take also m=·0034672.

Now let N_0, D_0 be the numerator and denominator of (62), with these values of h, m, and with Δ put equal to zero. Then, if $N=N_0+\delta N$, $D=D_0+\delta D$, $h=h_0+\delta h$ be the true values of those quantities, it is clear that

$$N=N_0+(1+\tfrac{18}{7}h+\tfrac{1}{7}m)\delta h,$$

$$D=D_0-\tfrac{2}{5}\sqrt{(1+\eta_1)}\delta h+\tfrac{1}{5}\frac{(1+h)}{\sqrt{(1-\eta_1)}}(\tfrac{5}{2}\frac{m}{h^2}-\tfrac{18}{7}-\tfrac{755}{14}\frac{m^2}{h^2})\delta h.$$

From these we may compute δN and δD.

I find then,

$$N_0=\cdot 0016160\text{8}, \qquad N=N_0(1+624\cdot 4\delta h),$$
$$D_0=\cdot 4991436, \qquad D=D_0(1+248\cdot 0\delta h).$$

Then $\frac{N_0}{D_0}=\cdot 0032377\text{1}$, and since the denominator should be augmented by 1·0003, it follows that the corrected value of $\frac{N_0}{D_0}=\cdot 00323674$, and

$$\frac{C-A}{C}=\frac{N}{D}=\cdot 00323674(1+376\cdot 4\delta h).$$

The most generally accepted value of the precessional constant is ·003272, and this exceeds our corrected N_0/D_0 by ·00003526. Therefore

$$\delta h = \frac{\cdot 00003526}{376\cdot 4 \times \cdot 0032366} = \cdot 00002895.$$

But h_0 was ·00332480, so that h = ·0033537. If $\frac{2}{14}h^2 + \frac{1}{7}f$ be added to h, we obtain e; the result is

$$e = \cdot 00333734 = \frac{1}{296\cdot 4}.*$$

I have, in fact, repeated the computation with this value of e, and find $\frac{N_0}{D_0} = \cdot 003273$, which only differs from the Precessional Constant by unity in the sixth place of decimals. Oppolzer † adopts the value ·003261 for the Precessional Constant, and this leads to $\frac{1}{297\cdot 5}$ as the value of the ellipticity.

Dr. Wiechert has considered the hypothesis that the Earth consists of an iron nucleus with a superstratum of rock. With the Precessional Constant at ·003272 it may be concluded from his table that the ellipticity is $\frac{1}{297\cdot 3}$.

These results, then, point to an ellipticity between $\frac{1}{296}$ and $\frac{1}{298}$, and they agree well with the results of all the methods of determining the ellipticity except that of the pendulum. Dr.

* As it is desirable that the ellipticity of the earth should be evaluated with all the accuracy possible, it may be well to advert to the augmentation of ellipticity which is due to the direct actions of the Moon and Sun.

The tide-generating potential of each of these bodies contains a term which is independent of the time, and to which there must correspond a permanent tide.

If i be the obliquity of the ecliptic; and m, c, e_0 the mass, mean distance and eccentricity of orbit for the Moon; while m', c', e'_0 represent the same for the Sun; the tide-generating potential contains the term,

$$\left[\frac{m}{c^3}(1 + \tfrac{3}{2}e_0^2) + \frac{m'}{c'^3}(1 + \tfrac{3}{2}e_0'^2)\right]\tfrac{3}{4}(1 - \tfrac{3}{2}\sin^2 i)r^2(\tfrac{1}{3} - \cos^2\theta).$$

This is equal to

$$(1\cdot 46035)\tfrac{3}{4}\frac{m}{c^3}(1 - \tfrac{3}{2}\sin^2 i + \tfrac{3}{2}e_0^2)r^2(\tfrac{1}{3} - \cos^2\theta).$$

The ellipticity of the Earth which corresponds with this term is

$$(1\cdot 46035)\frac{\frac{3m}{4M}\left(\frac{a}{c}\right)^3(1 - \frac{3}{2}\sin^2 i + \frac{3}{2}e_0^2)}{1 - \frac{3}{5}w/w_0} = \frac{\cdot 00000004708}{1 - \frac{3}{5}w/w_0}.$$

If we take $w_0/w = 2\cdot 1$, the ellipticity is ·00000006588. The meaning of this is that the Earth's surface is 28 cm. lower at the poles, and 14 cm. higher at the equator, than would be the case if the Sun and Moon were obliterated. This term may therefore be safely omitted.

† Helmert, *Höhere Geodäsie*, vol. ii. p. 437.

Helmert's* result from the pendulum is $\frac{1}{299{\cdot}26 \pm 1{\cdot}26}$, and is certainly slightly smaller than the value found here.

In the paper above referred to M. Callandreau has used the Precessional Constant for evaluating the ellipticity of the Earth, which he finds to be $\frac{1}{297{\cdot}4}$. The agreement of my work with his is thus very satisfactory.

Summary and Discussion.

The space in the neighbourhood of an oblate ellipsoid of revolution may be divided into three regions by two spheres touching it internally and externally. It is clearly possible to express the potential of such a solid homogeneous ellipsoid by series of spherical harmonics which are convergent both inside the smaller sphere and outside the larger one, but for the space between them the convergency is uncertain. In the treatment of the attractions of spheroids by means of spherical harmonics, it is usual to assume the ellipticity to be small, so that the region of possible divergency becomes negligible.

But this method is no longer certainly justifiable when we seek to carry the development as far as the squares of small quantities, and proof is needed of the applicability of the series within the middle region of space. If, having found our two series expressive of the potential for internal and for external space respectively, we determine the form of the surface inside the middle region at which these two potentials are continuous as far as the second order of small quantities, we find that the surface in question is that of the ellipsoid itself. The two series then form a continuous function at the surface of the ellipsoid, and they obviously satisfy the differential equation of the potential both inside and outside the ellipsoid. It follows that although the series were determined by a process which is open to doubt as respects the middle region, they lead to results which are trustworthy as far as the second order of small quantities.

There is, however, another method of finding the potential of the ellipsoid, by which the difficulty as to convergency is avoided. It is well known that the potential of a solid ellipsoid is expressible in a series of spherical harmonics, and that the series is convergent up to the surface, at least for all ellipsoids with eccentricity less than $\frac{1}{2}$. Also there is a rigorous expression in finite terms for the internal potential, involving only the second harmonic. If only the second power of the ellipticity be retained, these two expressions are found to be identical with those derived from integration, and the question of convergency is settled.

If an oblate ellipsoid of revolution be slightly distorted in any

* Vol. ii. p. 241.

way, the deformation being of the order of the square of the ellipticity, the additional terms in the potential may obviously be expressed by the ordinary formulæ of spherical harmonic analysis.

In the theory of the Earth's figure it is unnecessary to contemplate the existence of any other departure from true ellipticity than one expressible by a zonal harmonic of the fourth order. Three parameters are needed to express the surface of such a spheroid. If a and b denote the equatorial and polar radii, the first parameter is a the equatorial radius; the second is the ellipticity e defined as being equal to $\frac{a-b}{a}$; and it seems convenient to take as the third a quantity proportional to the elevation of the surface of the spheroid above that of the true ellipsoid in latitude 45°. If the third parameter be denoted by f, the elevation in question is $\frac{1}{4}af$.

I have found, however, that there is a gain in simplicity by displacing the ellipticity e from its position as the second parameter, and by substituting for it a parameter h, which is defined by

$$h=e-\tfrac{2}{1}\tfrac{5}{4}e^2-\tfrac{1}{7}f.$$

To the first order of small quantities it is clearly immaterial whether h or e be used, but the advantage of the change is found to arise when quantities of the second order are retained in the developments.

When the external and internal potentials of a homogeneous spheroid defined by a, h, f are known, it is easy to express by means of integrals the potential of a heterogeneous body built up by a succession of layers, each of which has its external and internal surfaces defined as a spheroid of the kind under consideration. When the object in view is the study of the figure of a planet, the potential corresponding to a uniform angular velocity must of course be added. The processes explained here are carried out in §§ 1 and 2.

It is generally assumed that the matter of which the Earth is formed is sufficiently plastic to permit the condition of hydrostatic equilibrium to be satisfied throughout the mass. Even if this condition is not rigorously correct, it must be nearly so.

In § 2 the condition of hydrostatic equilibrium is determined. It is expressed by two equations, of which the first corresponds to that ordinarily given for the ellipticity of internal strata, but it contains also terms of the second order. The second equation consists entirely of terms of the second order, and involves the parameter f. The advantage due to the substitution of h for e is now apparent, for the first equation is entirely independent of f. These two equations involve integrals which are eliminated by integration, and we obtain two differential equations of the second order for h and for f; the equation for h does not involve f (§§ 3, 4).

M. Radau has shown that it is possible to reduce the differential equation for the ellipticity to one of the first order, at least to a close degree of approximation. In § 5 his process is carried out with the retention of quantities of the second order. It appears that the approximation is even better than when only the terms of the first order are retained.

The formula for gravity is obtained in § 6. It was given for the first time by Airy, as already remarked in the Introduction. The results of pendulum experiments do not appear as yet to be sufficiently numerous or consistent, *inter se*, to render a revision of the value of the ellipticity of the Earth's surface practicable by aid of this more accurate formula for gravity.

The rates of change of h and of f at the earth's surface are determined in § 7. As a preliminary to the evaluation of the departure of the Earth's figure from true ellipticity, I then consider the figure of a homogeneous mass of fluid with a small heavy nucleus at its centre (§ 8). The departure is found to be greatest when the nucleus contributes ·457, and the fluid ·543 of the whole mass. For such a planet, with the same mean density, size, and length of day as the Earth, it is found that the surface is 8·8 metres below the true ellipsoid in latitude 45°. In the actual Earth the departure will certainly be less than this maximum.

The evaluation of f for a heterogeneous planet is only possible when the law of internal density is known. I have, then, adopted Roche's hypothesis, according to which the mean density of all the matter lying inside any surface of equal density is less than the central density by an amount which varies as the square of the equatorial radius of that surface. In the notation of the paper the law is expressed by $w_0=\rho[1-k(\frac{a}{a})^2]$. It is found in § 9, by some rather tedious analysis and computation, that the surface is depressed in latitude 45° by $3\frac{1}{4}$ metres.

Dr. Wiechert has maintained that the Earth probably consists of an iron nucleus with a rocky superstratum. His theory leads to the conclusion that the depression in latitude 45° amounts to 2·75 metres (§ 10). The close agreement between the results of such diverse hypotheses as those of Roche and of Wiechert appears to justify us in maintaining with confidence that the level surface is depressed below the true ellipsoid by 3 metres in latitude 45°.

A solution of the differential equation for h in § 11 does not lead to results of much general interest, and I refer the reader to that section for details.

It has been stated in the introduction that M. Callandreau has treated these problems by methods which differ somewhat from mine. He has concluded, but without definitely solving the differential equation, that the depression in latitude 45° must be less than 5 metres.

In § 12 formulæ are found for the moment of inertia of the

Earth about its axis of rotation, and thence for the Precessional Constant. Similar formulæ have been found by M. Callandreau in the papers referred to above, but I think that my formulæ are somewhat more succinct and tractable than his.

In the various theories of the figure of the Earth which have been propounded up to recent times, the value of the Precessional Constant has always been appealed to as the test whereby the correctness of the hypothesis as to internal density may be tried. But it appears from M. Radau's remarkable investigation that the Earth's moment of inertia about the axis of rotation is in reality nearly independent of the law of internal density; and the difference between the greatest and least moments of inertia depends rigorously on superficial data. Accordingly, the value of the Precessional Constant may be inferred with a considerable degree of accuracy from the form of the surface, and it affords evidence of little weight as to the law of internal density. But although from this point of view the comparison of this constant with theory becomes almost nugatory, yet the very result which shows its uselessness in one respect points out its utility in another, for we may now appeal to it as affording the means for an independent evaluation of the ellipticity of the Earth's surface.

In § 12 an estimate is made of the amount by which the moment of inertia of the earth is affected by the law of internal density. A formula is found for the moment of inertia, which consists of the sum of two parts, the first being dependent only on superficial data, and the second on the law of internal density. From the numerical table of results given in that section it appears that if the moment of inertia C be computed as though the second part were non-existent, it must be multiplied by the factor 1·0003 in order to take into account the neglected portion. Since the moment of inertia C occurs in the denominator of the Precessional Constant, it is obvious that the uncorrected value should be multiplied by ·9997. Proceeding from these results and from the value of the constant, it appears that the ellipticity of the Earth's surface must be about $\frac{1}{297}$. M. Callandreau has arrived also at the same conclusion, and it is confirmed by Dr. Wiechert, although his suggested law of internal density differs very widely from that adopted by other investigators. This estimate of the ellipticity agrees well with that derived from all the other methods, except that of the pendulum, from which it is concluded that the ellipticity is about $\frac{1}{299}$. It may be hoped that the various results may be brought into closer agreement with one another when the great mass of pendulum results now accumulated has been reduced.

It has been contended by Tisserand that the ellipticity of the Earth's surface is greater than any value which it is possible to reconcile with the existence of internal hydrostatic equilibrium; and with the values of the constants used by him, this is certainly so. But it seems to me that when the terms of the second order are included, and when more recent data are employed, there is

but little evidence in favour of this conclusion. If Tisserand were correct, it would indicate that the internal layers of the earth are more elliptic than is consistent with the present angular velocity of rotation. On the other hand, Dr. Wiechert seeks to show that his iron nucleus is deficient in ellipticity. His argument does not, however, carry conviction to my mind, as the data seem to be too uncertain for any such conclusion. It is, I think, preferable to maintain that nothing can, as yet, be decided on this point.

Note on the Values of the Coefficients of the terms of the Third Order in the new Lunar Theory. By Ernest W. Brown, M.A.

In the Memoirs of the Society, vol. liii. pp. 163–202, I have given the values of the coefficients of all terms of the third order with respect to the eccentricities, the inclination, and the ratio of the parallaxes; these values include all powers of the ratio of the mean motions since the numerical value of this ratio has been used. The results are given for rectangular co-ordinates. They also include the parts of the ratio of the mean motions, as far as these depend on the squares of the eccentricities, the inclination, and the ratio of the parallaxes. The values of the latter have been re-calculated since my paper of March 1897 * appeared, and no sensible differences from the values there given appeared. Instead of the value $-1739''\cdot6$ in the part of the motion of the perigee of the form $n\gamma_1^2 f(m)$ should be put $-1739''\cdot8$ (see p. 338 of the paper referred to); and for the value $-616''\cdot0$ in the part of the motion of the node of the form $ne^2 f(m)$ should be put $-616''\cdot1$ (p. 340). I may add that differences from Delaunay's values, after proper estimation had been made for the remainders of his series in powers of m, were found in the cases only of the parts of the motion of the perigee, which were of the forms $ne^2 f(m)$ and $ne'^2 f(m)$. The latter difference was found to be due to an error in Delaunay's theory, which was actually detected (*loc. cit.* pp. 340, 341). The former difference seemed to be due also to an error in Delaunay's theory in view of the three quite separate determinations which I have made, and which agree amongst themselves. If Delaunay's value had been correct, a difference from the observed value of about $26''$ in the annual motion of the perigee would have resulted. Hansen's and my values differ by less than $2''$ from the observed value. We must therefore conclude that in all probability Delaunay is in error.

These errors would naturally cause differences in the coefficients of $\sin(2i\mathrm{D}+l)$, $(i=\pm1, \pm2, \ldots)$ in longitude between Delaunay's theory and mine, as far as these coefficients depend on $e^3 m^j$, $ee'^2 m^j$. I have transformed my results to polar co-ordinates

* *Monthly Notices of R.A.S.*, vol. lvii. p. 332–341.

in order to compare them with those of Delaunay. Only one difference was found which seemed to indicate an error when estimation was made for remainders in Delaunay's results, and this difference was precisely where it might have been expected, namely, in the part of the coefficient of $\sin(2D-l)$, which is of the form $e^3f(m)$. The part of this coefficient, which is of the form $ee'^2f(m)$, is not carried sufficiently far in Delaunay's theory to notice any difference, as the series in powers of m converges very slowly. The part of the form $e^3f(m)$ in my theory contributes $-1''\cdot23$; in Delaunay's theory it is $-1''\cdot13$, or, when estimation is made for the remainder, $-1''\cdot07$. These small differences in the values of the coefficients of the periodic terms are, of course, not very important practically, but the comparison furnishes a valuable test of the accuracy of the various calculations, and in this respect there is no cause for dissatisfaction.

The terms of the fourth order are now nearing completion, and, so far as the various tests have been able to be applied, appear to fulfil the degree of accuracy which was aimed at. They have demanded more extended calculations than the terms of any other order have required or will require. Circumstances prevent them going forward very rapidly at present; but it is hoped that they may be ready for publication within the next twelve months.

Haverford College, Pa.:
1899 *November* 3.

On the proper Motions of Berlin B, Nos. 5072 *and* 5073.

By F. A. Bellamy.

In the comparison of Plate 1415, taken at the University Observatory, Oxford, for the *Astrographic Catalogue* in 1899, R.A. $14^h\ 20^m$, $+25°$, with stars in the *Astronomische Gesellschaft* Catalogues of Cambridge and Berlin, a large discordance was noticed between the Oxford measures and two stars Nos. 5072 and 5073 in the Berlin B Catalogue.

Failing to find any notification of an error or discordance in the *Bonn Durchmusterung* or Berlin B Catalogue, or any proper motions from the usual sources of information, we were fortunate to find that another plate of this region had been taken in 1893, but rejected through insufficient number of stars.

For comparison of this plate with 1415 four stars were selected and the measured co-ordinates compared. The linear corrections for reducing plate 399 to 1415 were found to be

$$+\cdot0083x+\cdot0013y-\cdot177 \text{ to } x.$$
$$-\cdot0013x+\cdot0083y-\cdot046 \text{ to } y.$$

To show how quickly two plates may be compared, the actual figures are given below (in reseau intervals).

					Berlin 5075	5072	5073
Plate 399 x	14·788	12·585	23·106	1·774	16·187	15·781	15·632
$-ax$	−·122	−·104	−·192	−·015	−·134	−·131	−·130
$-by$	−·003	−·033	−·016	−·020	−·002	−·003	−·003
$-c$	+·177	+·177	+·177	+·177	+·177	+·177	+·177
	14·840	12·625	23·075	1·916	16·228	15·824	15·676
Plate 1415 x	14·840	12·627	23·074	1·915	16·229	15·844	15·696
$x_{399}-x_{1415}$	−·000	−·002	+·001	+·001	−·001	−·020	−·020
Plate 399 y	2·413	25·193	12·440	15·506	1·697	2·093	2·051
$-dx$	+·019	+·016	+·030	+·002	+·021	+·020	+·020
$-ey$	−·020	−·209	−·103	−·129	−·014	−·017	−·017
$-f$	+·046	+·046	+·046	+·046	+·046	+·046	+·046
	2·458	25·046	12·413	15·425	1·750	2·142	2·100
Plate 1415 y	2·456	25·048	12·413	15·426	1·755	2·115	2·076
$y_{399}-y_{1415}$	+·002	−·002	·000	−·001	−·005	+·027	+·024

The differences for the two stars B 5072 and 5073 between the Oxford plates, which were exposed at epochs 1893·342 and 1899·400, are large, and undoubtedly due to proper motions which seem common to the two stars. From these differences annual Proper Motions of $+0^{s}\cdot070$ in R.A. and $-1''\cdot26$ in Dec. were deduced ; though the difference of epochs is only six years, these values may be very near the truth, as they depend solely on measures from two photographic plates, and not on meridian observations.

Using these proper motions, and carrying back the Oxford measures to the epoch of Berlin, we get the following comparison :—

			s	″
Berlin B. 5072	Epoch 1880·9.	Oxf.—Berl.	−0·07	and +0·9
Berlin B. 5073	Epoch 1881·1.	Oxf.—Berl.	−0·04	„ +0·6

The region which includes these two stars has also been photographed at the Paris Observatory ; and in the Report for 1895, page 18, the plate, 954, is mentioned as having been measured.

The positions for the two stars for 1900·0 from Oxford measures are

		h	m	s	°	′	″
Berlin B.	5072	14	21	5·40	+24	5	55·2
	5073	14	21	8·59	+24	6	7·5

I have not been successful in finding any observations of these two stars as a double star, probably the wide distance causing them to escape notice ; moreover they are both fainter than the 9th magnitude, and possibly 5073 is slightly variable, as it is not given in B.D.—though noted as 9·1 at Berlin—unless only the brighter of the two was purposely observed. But their large common proper motion seems to show that they are physically connected. The stars deserve to have some observations made for magnitude, motion in space or about each other, and for parallax.

During the course of work for the *Astrographic Catalogue* at the University Observatory, Oxford, many occasions have arisen when plates taken in the earlier years have been of great use in checking, if not in determining, proper motions.

Had the zones $+25°$, $+27°$, $+29°$, $+31°$, been systematically photographed in the earlier period of the work, and the intermediate or overlapping zones, $+26°$, $+28°$, $+30°$, been left till the later years, we should have had at Oxford a difference of epoch of not less than five years, quite sufficient to, at least, verify proper motions. This, however, has not been done here ; but, as early plates, rejected through want of stars, often become valuable for this purpose, it may be useful to bear this in mind at those observatories where the plates have not yet been taken. Dr. Gill's proposal in a recent report * to take all the plates over again for measurement seems an excellent one.

University Observatory, Oxford :
1899 *November* 22.

Since writing the above paper I have recollected that the late Radcliffe observer, Mr. E. J. Stone, published a paper † on a case somewhat analogous to this. The paper refers to the proper motions of four stars in the southern hemisphere, ζ *Toucani*, ζ^1 and ζ^2 *Reticuli*, and ϵ *Eridani*. From his discussion Mr. Stone concluded that the four stars have a common proper motion of more than $1''$, also that each star is moving away from every other star in the group, being at one time near to each other. The four stars are widely separated in N.P.D., being $2\frac{1}{2}°$, $4'$, $19\frac{1}{2}°$! respectively ; and in R.A. ζ *Toucani* is 3^h ! from the other stars, giving an angular distance of $33°$!

As these four stars fall in zones photographed for the *Astrographic Catalogue* at the Cape of Good Hope and Melbourne Observatories, it is hoped that they will not be neglected, but a further discussion be made with recent material.

November 24.

* *Monthly Notices*, lix. p. 247.
† *Monthly Notices*, xl. p. 26.

New Nebulæ discovered photographically with the Crossley Reflector of the Lick Observatory. By James E. Keeler, D.Sc.

In a recent number of the *Monthly Notices* (vol. lix. p. 537) I printed a list of seven small nebulæ found on plates which had been exposed to the nebula *Messier* 51 with the Crossley reflector. At that time I considered this a rather remarkable number of nebulæ to be found on a plate covering only about one square degree, but more recent experience has shown that as many new nebulæ may generally be found on photographs taken with the Crossley reflector, and sometimes a much larger number. Thus, on a plate exposed for four hours, on November 6, to the nebula H V. 19 *Andromedæ*, there are thirty-one new nebulæ and nebulous stars; on a photograph of H I. 53 *Pegasi*, twenty; and there are nearly as many on several other plates. Besides these new nebulæ, the existence of which has been verified by independent photographs, the plates contain a considerable number of objects which are probably nebulæ so small that the resolving power of the telescope is insufficient to define them in their real form and to bring out their true character. They appear as star-discs which are less dense than true star-discs having the same diameter.

With exposures of four hours the Crossley photographs show stars and nebulæ far beyond the range of any visual telescope, though there are hundreds, if not thousands, of unrecorded nebulæ within reach of our 36-inch refractor. On the assumption that there are three new nebulæ in each square degree (a number which is much below the average for the photographs I have already taken), the number of new nebulæ in the whole sky would be about 120,000. Doubtless there are large regions where few, if any, new nebulæ are to be found; still, I am disposed to regard the above estimate as below the truth.

It is a remarkable and highly significant fact that most of the nebulæ photographed with the Crossley reflector seem to be spirals. In various cases known nebulæ have already been shown to have a spiral form by other observers. The Crossley photographs have shown that many other known nebulæ have the same structure; while in the case of a large proportion of the new nebulæ I have mentioned, a spiral form is either directly recognisable, or is rendered highly probable by the fact that the nebulæ have the same appearance as well-known spirals when photographed with a small instrument. We may perhaps regard the spiral form as that which is normally assumed by a compact isolated nebulous mass, and the exceptions (of which there are many) to somewhat unusual conditions—possibly, in part, to the absence of any definite moment of rotation for the aggregate of the particles constituting the nebula.

Lick Observatory, University of California:
1899 *November* 15.

Nebulæ discovered at the Chamberlin Observatory, University Park, Colorado. By Herbert A. Howe.

(*Communicated by the Secretaries.*)

The following nebulæ have been incidentally noted during the past few months, while making measures of catalogued nebulæ with the 20-inch refractor. The positions given depend upon micrometric measures, and are for 1900·0. In the "Descriptions" and "Notes" numbers enclosed in brackets refer to Dreyer's Index Catalogue; other numbers are those of the N.G.C. :—

No.	Date. 1898.	R.A. h m s	Dec. ° ′	Descriptions.
1	Dec. 16	0 50 50	−10 31·6	vF, S, near 309.
2	1899. Jan. 6	2 29 16	−11 28·7	eS, vF, R, prob. nebs. ✶. Near 977 and 981.
3	Mar. 14	9 22 9	−11 40·5	F, vS, 10^m ✶ p 7^s, 0′·8 n.
4	Apr. 10	10 31 20	−12 12·3	eF, eS, possibly a ✶.
5	May 11	12 43 46	−13 51·1	vF, vmE 210°, 2′ long.
6	May 10	12 45 2	−13 50·1	eF, vS.
7	May 10	12 45 9	−13 46·6	eF, vS.
8	May 10	12 45 16	−13 52·9	eF, vS.
9	May 10	12 45 23	−13 56·9 ±	eeF, vS. possibly a 14^m ✶.
10	May 11	12 46 4	−14 1·8	F, vS, R.
11	May 10	14 14 12	− 4 1·6	vF, S, mE 200°. Near (997).
12	1898. Sept. 7	18 36 40	+39 56·1	eS, eF. Near 6685 and 6686.
13	Sept. 17	21 34 6	−22 51·4	vS, eF. Near 7103 and 7104.
14	Oct. 11	23 34 19	−22 58·0	vF, vS, R, 6′·5 n of Swift 234.
15	Nov. 16	23 41 10	−28 33·6	vF, vmE 200°, 20″ long.

Notes.

No. 1 precedes 309 51^s, 1′·5 south.

No. 3 follows 2881 about a minute (of time).

No. 4 is near 3295 and 3296, which precede the places given by Leavenworth by 2^m 40^s. Leavenworth gave the same right ascension for 3295, 3296, and 3297. On 1899 April 10 I measured the places of 3295 and 3296. I could only suspect 3297. The object which I have supposed to be new follows the others 3^m 30^s.

Nos. 5–10, together with No. 15 of my former list, published in *Monthly Notices*, vol. lviii. No. 9, are in the vicinity of 4724 and 4727. I believe 4726 and 4740 to be identical at 12^h 46^m 18^s, −13° 4′·6.

No. 11 precedes (997) about 30^s. (998), which its discoverer pronounces "eeF, v diffic.," I did not see. In its position, or very near it, is a double star of mags. 13·5–13·5, distance 30″, and angle 160°.

No. 12 precedes 6685 less than 2^s, 2′·7 north. Between them lies a star of mag. 11·5.

No. 13 is in the same field with 7103, 7104 (1393), and No. 17 of my list in *Monthly Notices*, vol. lviii. No. 9. These five objects have all been seen in one night. Between my two novæ and 7103 one or two may exist, having

been suspected on more than one night. A large telescope may well deal with this group.

No. 15 has puzzled me somewhat. The southern end is the brightest portion, and at times it seems as if the object were really a very faint double star, one or both components of which are nebulous.

Observations of Nebulæ made at the Chamberlin Observatory, University Park, Colorado. By Herbert A. Howe.

(*Communicated by the Secretaries.*)

The following notes on nebulæ form a continuation of those given on pp. 515–24 of the Supplementary Number of the *Monthly Notices* for 1898. They were made during the twelve months beginning 1898 July 1 and ending 1899 June 30. During this period illness and occupation with other observations, especially upon *Eros*, have notably interfered with the regular observations of nebulæ.

The numbers given below are the current ones of the N.G.C., except those which are enclosed in brackets, which refer to the Index Catalogue in vol. li. of the *Memoirs of the R.A.S.* When the name Swift is given, followed by a number, reference is made to the list published in *A.N.* 3517. In this list Swift has collected all previous discoveries of nebulæ at the Lowe Observatory, and has numbered them consecutively. Positions of a few Marth nebulæ are given below, because those in the N.G.C. are only approximate, though they are not far astray. Data about position angles and distances found in the following notes are, in the main, not derived from measures, but from eye-estimates. I have frequently made special notes about condensations in faint nebulæ, where it has seemed that the condensations were sufficiently bright and well defined to be suitable for measures of parallax with a large telescope. All positions are referred to the mean equinox of 1900·0. The eyepiece used on the new Bruce micrometer magnifies 200 diameters, and gives a field 15′ in diameter.

As previously, all observations were made with the 20-inch Clark-Saegmuller equatorial refractor.

Swift 1. The position is $0^h\ 1^m\ 22^s$, $-4°\ 16'{\cdot}4$.

135. The position is $0^h\ 26^m\ 43^s$, $-13°\ 53'{\cdot}3$.

178. The position is $0^h\ 34^m\ 7^s$, $-14°\ 43'{\cdot}2$.

209. This may almost be called a nebulous star. Its position is $0^h\ 34^m\ 4^s$, $-19°\ 9'{\cdot}4$.

232. The position is $0^h\ 37^m\ 48^s$, $-24°\ 6'{\cdot}5$.

235. The position is $0^h\ 37^m\ 56^s$, $-24°\ 5'{\cdot}4$.

303. This is elongated at 160°. Its position is $0^h\ 49^m\ 56^s$, $-17°\ 11'{\cdot}8$.

309. The position is $0^h\ 51^m\ 41^s$, $-10°\ 27'{\cdot}2$.

333. The position is $0^h\ 53^m\ 54^s$, $-17°\ 0'{\cdot}5$.

Swift 24. This looks resolvable, and is equivalent in bright-

ness to a star of mag. 10–11. It is very small. The position is $1^h\ 43^m\ 19^s$, $-27°\ 23'·3$.

686. In the N.G.C. is the note "2 st nr." They are of mags. 7 and 8, and precede the nebula.

808. The elongation is at 200°.

921. The position is $2^h\ 21^m\ 48^s$, $-16°\ 17'·8$.

929. The position is $2^h\ 22^m\ 28^s$, $-12°\ 32'·1$.

942–3. Of this very faint double nebula the southern component is the brighter, being equivalent to a star of mag. 13. The northern is of mag. 13·5. The angle is 350°, and the distance 40″. Both have good nuclei. Their positions were given in *Monthly Notices*, vol. lviii. No. 6. On one night an eF nebula was suspected 20 seconds preceding this pair.

944. This is much elongated at 210°, and looks like a nebulous double star. The position is $2^h\ 21^m\ 54^s$, $-14°\ 58'·0$.

981. The position is $2^h\ 28^m\ 9^s$, $-11°\ 24'·9$.

989. The position is $2^h\ 29^m\ 3^s$, $-16°\ 57'·0$.

1013. This has a good nuclear brightening, and would be suitable for parallax measures with a large telescope.

1074. The position is $2^h\ 38^m\ 54^s$, $-16°\ 43'·3$.

1075. This contains a condensation of mag. 13·5. Its position is $2^h\ 38^m\ 52^s$, $-16°\ 37'·5$. It is to be noticed that 1075 really precedes 1074. The N.G.C. gives the same right ascension for both.

1151. Leavenworth queried whether this were a nebula. It seemed to me not to be stellar, as I noted it as "eF, vS."

1180 and 1181. Leavenworth gave the same right ascension for these, the southern one being 1181. It has a good nucleus of mag. 13·5. To avoid confusion the N.G.C. numbers should be retained, though 1181 really precedes 1180. The positions are :—

1180	... ...	$2^h\ 52^m\ 22^s$, $-15°\ 24'·8$.
1181	... ...	$2^h\ 52^m\ 19^s$, $-15°\ 26'·9$.

1182. The nucleus is of mag. 14. The position is $2^h\ 58^m\ 38^s$, $-10°\ 3'·7$.

1413. This is almost stellar, so faint is the nebulosity surrounding the central eS condensation. The position is $3^h\ 35^m\ 35^s$, $-15°\ 56'·0$.

1416. Muller gave this nebula as 2′ north of a star of mag. 8·6. It is really south of the star. There is another star of equal mag. about 5′ south of the star mentioned. The position of the nebula is $3^h\ 36^m\ 41^s$, $-23°\ 2'·4$

1445. The position is $3^h\ 40^m\ 9^s$, $-10°\ 10'·4$.

1486. The position is $3^h\ 51^m\ 57^s$, $-22°\ 6'·7$.

1547. Leavenworth queried whether this were a cluster. I had no such suspicion. The southern end is the brightest portion. The position is $4^h\ 12^m\ 44^s$, $-18°\ 6'·3$.

1561–5. In my communication in *Monthly Notices*, vol. lviii. No. 9, it was stated that I could not see all these nebulæ. I have finally, however, been able to find and measure nebulæ which correspond to Leavenworth's descriptions of those in this group. His positions are sufficiently erroneous to warrant the publication of the correct places. To this group I have added one new nebula, the place of which was published on p. 523 of vol. lviii. of the *Monthly Notices*. I suspect another eF nebula near 1565. Its estimated position is $4^h\ 18^m\ 54^s \pm$, $-15°\ 55' \pm$. The positions are as follows :—

	h	m	s	°	′
1561	4	18	28,	−16	4·7
1562	4	17	15,	−15	59·5.
1563	4	18	21,	−15	57·9.
1564	4	18	28,	−15	58·3.
1565	4	18	51,	−15	58·6.

It is to be noted that 1562 precedes the rest of the group over a minute. According to Leavenworth it is brighter than 1563, 1564, or 1565 ; no nebula brighter than these exists in the place given by Leavenworth for 1562. As usual, Leavenworth's declinations are much closer to the truth than his right ascensions.

1650. Two nebulæ were suspected near this, one np and the other sf, at distances of about 3′. The np one is probably only two eF stars. The position of 1650 is $4^h\ 40^m\ 40^s$, $-16°\ 3'·2$.

1821. The elongation is at 300°. The position is $5^h\ 7^m\ 14^s$, $-15°\ 15'·4$.

Swift 80. Swift says : "bet 2 stars p and f." I see no stars in this position relative to the nebula, which are sufficiently near to aid in identifying it. The position is $5^h\ 27^m\ 52^s$, $-17°\ 17'·7$.

2139. For this I searched on two nights unsuccessfully, though H calls it only "F." I believe it to be identical with Swift 90, which is about 10′ north, and 25ˢ following.

Swift 90. This seemed to me to be fainter and smaller than the description "B,L" would indicate. The position has been given in *Monthly Notices* lviii., No. 9.

Swift 91. This is bush-like, and seems to contain a star of mag. 14. The position is $6^h\ 1^m\ 22^s$, $-27°\ 50'·9$.

2263. The two stars mentioned by h are of mags. 11·5 and 12, and lie respectively north and south of the nebula at distances of, roughly, 2′, slightly following. Another star of mag. 11·5 follows the nebula 1′·7, and is 0′·3 north.

2283. This curious object, discovered by H, is a small quadrilateral of stars of mags. 11, 12, 11 and 13, the interior of which is nearly filled by an eF nebulosity, in

which there is a condensation of mag. 14. This condensation lies at about the middle point of the sp side of the quadrilateral.

Swift 92. This is in a pretty rich field of faint stars. In the centre of the nebula is a condensation of mag. 13.

2612. h calls this "B." I make it only F. The elongation is at 100°. The two stars mentioned by h are of mags. 10 and 11. The brighter star lies 1′·2 nearly due south of the nebula. The fainter is nearer, north, and a little following.

2674. For this I have hunted on two nights in vain. Stone describes it as "eF, S, neb. ?"

2706. The elongation is at 170°, the nebula being about 1′ long and one-fourth as broad. The neighbouring star noted by Swift is of mag. 9, and follows 5ˢ, 0′·5 south.

2757. Muller queried whether this eF object were a star. The longer I looked at it the more positive I became that it was a double star of mags. 14, angle 45°, and distance 12″.

2811. This has a small bright centre, with faint wings at angles of 210° and 30°, and reminds one of the great nebula in *Andromeda*.

2821. At times this seemed to have a nucleus of mag. 13·5. h speaks of a "* 11 att." It is np the nebula, and I saw no connection between them.

2881. The two stars mentioned by Swift are of mags. 9·5 and 10·5, and are sf the nebula.

Swift 95. The two stars mentioned by Swift are of mags. 9 and 9·5. The brighter follows the nebula 3ˢ, 1′·3 south. The fainter stands at a position angle of 60° with reference to the brighter, and is about 1′·5 distant from it. The position is $9^h\ 31^m\ 16^s$, −11° 59′·4.

3030. The position is $9^h\ 45^m\ 18^s$, −11° 45′·4.

3058. The following nucleus is the brighter. The preceding is at 210°, 20″. The position of the brighter nucleus is $9^h\ 48^m\ 44^s$, −12° 0′·7.

3113. The nebula follows the two stars of mag. 8 mentioned by h as forming a triangle with it. The triangle is rudely equilateral. The nebula is eF and vL, but seems to have one spot a trifle brighter than the rest.

3280 and 3295. I found nothing in the N.G.C. position for 3295. But a nebula which corresponds to Leavenworth's description was found less than 3^m preceding, at nearly the N.G.C. declination. It involves a wide double of mags. 13·5, distance 30″, and angle 70°. 3280, discovered by Common, has a description similar to that of 3295. As the positions of Common's nebulæ are not very accurate, I judge 3280 and 3295 to be the same. The position is $10^h\ 27^m\ 48^s$, −12° 7′·3.

3296. The large error of the N.G.C. right ascension of this

object is almost exactly the same as the corresponding error for 3295, which was discovered by the same observer. The position of 3296 is $10^h\ 27^m\ 50^s$, $-12°\ 12'{\cdot}1$.

Swift 112. The doubles mentioned by Swift are very wide, and the average magnitude of the stars in them is 11. They are np and sf the nebula. Swift probably made an error of 15′ in reading his declination circle for this object. The position is $10^h\ 31^m\ 19^s$, $-23°\ 48'{\cdot}3$.

3704. I have now hunted in vain for this object on four nights, widely separated. On each night I measured the position of its neighbour 3707, which is supposed to be of the same order of brightness, and which is not at all difficult to see. Both are Tempel nebulæ.

3777. The position is $11^h\ 31^m\ 3^s$, $-12°\ 0'{\cdot}8$.

3831. h calls this "R," while I find it to be elongated at 200°.

3969. This I was unable to find in the N.G.C. place. But about 10′ further south I found one answering to its description, except that Stone notes "* 10 np 4′," while I see a star of mag. 8·5 nearly north. The stellar point in the centre of the nebula is of mag. 14. The position is $11^h\ 50^m\ 3^s$, $-18°\ 22'{\cdot}2$.

Swift 131. The elongation is at 120°.

4225. H. says, "* 170°, 60″." The star referred to is of mag. 8·5, follows the nebula $0^s{\cdot}6$, and is 1′·6 south of it.

4279. Swift rates this at the same brightness as 4280. But I could not find it on either of two nights, though it is supposed to be close to 4280 and 4285, both of which I saw. There is nothing there comparable in brightness to 4280 and 4285.

4285. This was noted on one night as fainter and smaller than 4280. On another night it was called "much fainter" than 4280. But Swift describes it as brighter.

4361. This has a vB sharply defined eS nucleus of mag. 9·5. On account of moonlight I could not see much surrounding nebulosity.

Swift 138. The star of mag. 7 mentioned by Swift follows the nebula 18^s, 2′·6 north. The position of the nebula is $12^h\ 19^m\ 44^s$, $-25°\ 28'{\cdot}6$.

4484. The N.G.C. right ascension of this seems to be 2^m in error. The position is $12^h\ 23^m\ 43^s$, $-11°\ 5'{\cdot}9$.

Swift 143. The position is $12^h\ 44^m\ 42^s$, $-25°\ 22'{\cdot}6$.

4724, 4726, 4727, and 4740. In my communication in *Monthly Notices*, vol. lviii. No. 9, I mentioned the region in which these lie as needing exploration with a large telescope because of the errors in the places of 4726 and 4740. A larger telescope than mine might show many more nebulæ in this group. The results of my work on three nights are as follows:—The N.G.C. positions of 4724 and 4727 are practically correct. I can find nothing

in the N.G.C. place for 4726. On two nights I measured the position of an object which I assume to be 4726 : it is at $12^h\ 46^m\ 18^s$, $-13°\ 40'·6$. I cannot see anything in the N.G.C. place for 4740. Under date of 1899 August 9 Dr. Swift writes, in reply to a query : "I have examined the record of 4740, and find that it was made 1887 April 27, with position $12^h\ 46^m$, $-13°\ 41'$. The right ascension is for 1890, the declination for date of discovery. I have no recollection about it." This position agrees so well with that of 4726 that I assume them to be identical.

I have discovered in this vicinity seven objects supposed to be new. Their descriptions and positions follow :—

The first Nova is vF, vmE 210° 2′ long. The position of its brightest part is $12^h\ 43^m\ 46^s$, $-13°\ 51'·1$.

The second Nova is eF, vS : its position is $12^h\ 45^m\ 2^s$, $-13°\ 50'·1$.

The third Nova is eF, vS : its position is $12^h\ 45^m\ 9^s$, $-13°\ 46'·6$.

The fourth Nova is eF, vS : its position is $12^h\ 45^m\ 16^s$, $-13°\ 52'·9$.

The fifth Nova is eeF, vS. Possibly it is only a star of mag. 14. It precedes the sixth Nova by 15^s, at nearly the same declination. Its position therefore is $12^h\ 45^m\ 23^s$, $-13°\ 56'·9\pm$.

The sixth Nova is vF, vS, R, 0′·6 n of star of mag. 11. Its position is $12^h\ 45^m\ 38^s$, $-13°\ 56'·9$.

The seventh Nova is F, vS, R : its position is $12^h\ 46^m\ 4^s$, $-14°\ 1'·8$.

4792. While measuring 4794 I hunted for this, which Tempel calls "vS, R, 7′ nnp 4794." I could not be sure of it. Possibly it is a suspicious-looking star of mag. 11.

4794. The nebula contains a condensation of mag. 13. The brightest of the stars mentioned by H is of mag. 9·5, and follows the condensation $1^s·2$, 3″ north. This star appears to be close to the nf border of the nebula.

5420. The position is $13^h\ 58^m\ 36^s$, $-14°\ 8'·1$.

Swift 168 and 169. Probably these are identical with (997) and (998). I examined the region on one night only, and saw only a very faint double star in or near the place of (998).

6230. This nebula is accompanied by a star of mag. 13, at a distance of 10″ and angle of 160°.

6235. In the N.G.C. this is given as "r r r, st 14–16." I examined it when the seeing was excellent. The brightest star is of mag. 12, and is in the np end. The object seems nebulous, but contains a number of very faint stars. Possibly the nebulous-appearing background is composed simply of minute stars, but it impressed me as truly nebulous.

6284 and 6287. The N.G.C. descriptions of these differ but

little. Each is described as a cluster. I examined them on the same night, and noted that 6284 was probably a nebula, while 6287 seemed to be a cluster. Perhaps greater optical power is necessary for a decision.

(1247). This nebula of Bigourdan's I was unable to find. 30^s preceding the place given in the Index Catalogue there is a double star of mags. 12—12, distance 3″ and angle 270°.

6369. In this annular nebula the centre is much darker than the periphery. The brightest part is in the np portion of the ring. The faintest part is in the sf portion of the ring.

6432. This "cluster" contains only four stars, two of mag. 12 and two of mag. 13.

6445. The appearance of this is remarkable. It is somewhat bullet-shaped. The sf end seems to be cut almost straight across, the position angle of its direction being 70°. The brightest part of the nebula is in the np end. The middle is much fainter than the ends.

6449. In this are two or three stellar points.

6450. This was searched for unsuccessfully on two good nights.

6465. Search was made for this on two nights. On the first no nebula could be found. On the second it was discovered that, instead of a nebula, there are simply two doubles of mag. 12. In each pair the distance is 4″, and the two pairs are 15″ apart.

6530. In the N.G.C. this is described as following 6523. It is really involved in faint nebulosity on the borders of 6523.

(1271). An examination on two nights led to the conclusion that it is simply an eF extension of 6523.

6544. In the N.G.C. this is called "r." I examined it on two nights. It evidently contains several stars. On the first night it was noted that "the apparent nebulosity may be due to faint stars." On the second night, when the definition was fine, the following note was made: "Object looks exclusively starry." If these observations are correct it is simply a faint cluster.

6551. Here I see simply a few stars of mag. 13.

(1274). Here is a nebulous region looking like a thin veil over the sky.

6556. I see nothing in the entire region except thousands of the minutest stars.

6559. The nebula itself is very faint and formless. The brightest star in it is of mag. 8·5, and has a 10·5 mag. companion at a distance of 6″ and angle of 190°.

6589. The double star mentioned by Swift as involved in the nebula is of mags. 13-13, and has a distance of 20″, with an angle of 210°. 5^s preceding the nebula is another

double of mags. 9–11·5, having a distance of 30″ and an angle of 225°.

6593. This is eS, and about as bright as a star of mag. 12. Very little nebulosity is visible.

6616. The "2 F st nr" are of mags. 9 and 10 respectively. The brighter precedes the nebula 2^s, 0′·6 south. The position is $18^h\ 13^m\ 28^s$, +22° 12′·0.

6618. This contains many very faint stars; changes of brightness in different parts are very abrupt. Dark "holes" are near by. The brightest part is small, elongated at about 165°, and contains some extremely faint stars. A magnificent object.

6642. The outskirts of this can be resolved. The bright centre I think to be stellar.

6660 and 6661. A careful examination of this region confirms Pechüle's statement that they are identical. The correct position is $8^h\ 30^m\ 25^s$, +22° 49′·8.

6814. This has a nucleus of mag. 13.

6822. On two nights I called this "vS," while Barnard, who discovered it with a 6 inch refractor, called it "L."

(1308). A double star follows 5^s, 40″ south. It is of mags. 9–12, distance 6″, and angle 150°.

6836. In this nebula, or just on its f edge, is a star of mag. 13·5, apparently not noticed by its discoverer.

Swift 185. The three stars mentioned by Swift are of mags. 9·5, 10, and 9, and are about 8′ south of the nebula. Their line prolonged does not strike the nebula, but follows it a little. The position is $19^h\ 50^m\ 10^s$, −37° 35′·6.

Swift 186. The position is $19^h\ 53^m\ 32^s$, −38° 50′·9.

Swift 187. The position is $19^h\ 54^m\ 10^s$, −38° 50′·8.

6903. h calls this "cL"; I observed it on one night, and noted it as "eS."

(1324). The position is $20^h\ 26^m\ 48^s$, −9° 23′·6.

(1325) and (1326). I am inclined to the opinion that these two Swift nebulæ are identical with two which Marth found with the 4-foot Lassell reflector, mounted at Malta, and which are Nos. 6928 and 6930 respectively in the N.G.C. Marth seems to have discovered these, as well as 6927, on the same evening, and afterwards verified them. It appears improbable that Swift, who makes no mention of seeing Marth's nebulæ, has discovered two close by them, which I did not see. A star of mag. 11 is just nf 6928, and a double star of mags. 12, and distance 5″ follows 6928 by 20^s, 0″·7 north. The positions are:—

6928, $20^h\ 28^m\ 1^s$, +9° 35′·2
6930, $20^h\ 28^m\ 10^s$, +9° 32′·0

When observing these objects I was unaware of the existence of 6927, which is much fainter, and which I did not see.

Swift 203. The position is $20^h\ 59^m\ 3^s$, $+11°\ 22'{\cdot}0$.

(1368). Swift calls this round, but I find it to be much elongated at 225°.

7105. On three nights I failed to find this. On the third night I noted in its vicinity a double of mags. 13–13, distance 8″, and angle 210°. This might be 7105, except that Leavenworth estimated the elongation of his at 130°.

7112. A search on two nights failed to reveal this.

Swift 207. The position is $21^h\ 41^m\ 25^s$, $-35°\ 20'{\cdot}8$.

Swift 208. The position is $21^h\ 42^m\ 20^s$, $-35°\ 24'{\cdot}9$.

7135. In the N.G.C. is noted a "✶ 14 att. p." This is of mag. 12, and precedes 2^s. Another star of like brightness is involved in the nebula near its northern border. I judge this to be identical with Swift 209. The object which I saw answers to both descriptions.

7152. According to the N.G.C., Lassell did not succeed in finding this. It is a small, exceedingly faint, and diffuse stain on the sky.

7157. In the N.G.C. we read for this object, "BD ✶ p 8^s." There is no bright double star in the vicinity, and I could find no nebula. Possibly, however, the abbreviation "BD" is here used for "Bonn Durchmusterung." Spitaler also failed to find this.

7158. This nebulous star is of mag. 13. It may be double at 270°. The position is $21^h\ 52^m\ 6^s$, $-12°\ 4'{\cdot}1$.

7165. This nebula contains a condensation of mag. 13.

7188. The position is $21^h\ 57^m\ 57^s$, $-20°\ 48'{\cdot}1$.

7214. h calls this a globular cluster. To me it appeared to be a nebula having a nuclear condensation of mag. 12 ; the surrounding nebulosity was indefinite in extent.

7247. The position is $22^h\ 12^m\ 7^s$, $-24°\ 13'{\cdot}8$.

7252. This is small and pretty faint, but has a good nucleus.

Swift 213. The position is $22^h\ 17^m\ 4^s$, $-19°\ 22'{\cdot}4$.

Swift 214. The position is $22^h\ 17^m\ 16^s$, $-19°\ 23'{\cdot}1$.

7287. This is a double star of mags. 11·5–11·5, distance 6″ and angle 150°. The seeing was poor when I examined it ; I could not see any nebulosity connected with it. The right ascension given in the N.G.C. is about 2^m too large.

(1447). The position is $22^h\ 24^m\ 48^s$, $-5°\ 38'{\cdot}0$.

7294. The position is $22^h\ 26^m\ 36^s$, $-25°\ 54'{\cdot}7$.

7300. In the N.G.C. this is called "cS." There is, however, considerable outstanding nebulosity which entitles it to the appellation "pL." The elongation is at 150°.

7310. The position is $22^h\ 29^m\ 9^s$, $-23°\ 0'{\cdot}0$.

7313. The position of this Marth nebula is $22^h\ 30^m\ 0^s$, $-26°37'{\cdot}1$.

7314. This is roughly estimated to be $2'{\cdot}3$ in length, and $0'{\cdot}8$ broad, the elongation being at 0°. There is a condensation of mag. 12 below the middle, near the preceding

edge. This condensation looks just like a star, and probably should be thus denominated.

7351. Swift calls this "R," but to me it appeared much elongated at 180°.

7359. The right ascension given in the Notes at the end of the Index Catalogue, and also that in the N.G.C., are erroneous. The nebula has a central condensation of about 11·5 mag., which is a trifle elongated. The position is $22^h\ 39^m\ 21^s$, $-24°\ 12'·8$.

7365. The position is $22^h\ 39^m\ 47^s$, $-20°\ 28'·6$.

7399. The position is $22^h\ 47^m\ 25^s$, $-9°\ 47'·9$.

7413. The position is $22^h\ 50^m\ 4^s$, $+12°\ 41'·4$.

(1461). I could find nothing in the N.G.C. place for this. But 10′ farther north there is a nebula corresponding fairly to Swift's description. It is, however, eS, being almost stellar, and of about mag. 12. I judge this to be Swift's object. Its position is $22^h\ 53^m\ 37^s$, $+14°\ 38'·3$.

7452. I suspected another nebula preceding this about 15^s, but the atmospheric conditions were not of the best.

7494. The position of this Marth nebula is $23^h\ 3^m\ 36^s$, $-24°\ 54'·7$.

7498. The position of this Marth nebula is $23^h\ 4^m\ 35^s$, $-24°\ 58'·0$.

7513. The position of this Marth nebula is $23^h\ 7^m\ 51^s$, $-28°\ 54'·0$.

7520. On two nights I have searched for this nebula of Tempel's without success.

Swift 229. In Swift's first list of nebular discoveries made at the Lowe Observatory, which one may find in No. 388 of the *Astronomical Journal*, this nebula is put at −18° of declination. In his later long list in *A.N.* 3517, the declination is given as −19°. I have found an object agreeing with his in description at $23^h\ 21^m\ 5^s$, $-18°\ 30'·3$. This place differs from the one in the *Astronomical Journal* by 5^s in right ascension and 5′·7 in declination. In *A.N.* 3517 the description says, "F ✻ p close nf." In the *Astronomical Journal* the star is said to *precede*. The object which I observed had a star of mag. 10 preceding it 3^s, 0′·6 north. [*Note.*—Dr. Swift has just looked up his original record for me; it is now evident that both the *Journal* and *Nachrichten* were wrong, and that the object which I found was really Swift 229.]

Swift 231. The position is $23^h\ 24^m\ 17^s$, $-29°\ 23'·0$.

7719. The position is $23^h\ 32^m\ 49^s$, $-23°\ 31'·6$.

Swift 234. The position is $23^h\ 34^m\ 9^s$, $-23°\ 3'·1$.

7730. On two nights I searched for this nebula, which Tempel called "pB, pL," but I could not find it.

(1505). The position is $23^h\ 36^m\ 29^s$, $-4°\ 7'·1$.

Swift 236. The position is $23^h\ 42^m\ 2^s$, $-28°\ 30'·7$.

Swift 237. The position is $23^h\ 42^m\ 16^s$, $-28°\ 39'·8$.

Swift 238. The position is $23^h\ 42^m\ 16^s$, $-28°\ 41'{\cdot}4$.

Swift 239. This is binuclear, at an angle of 250°, with a distance of 20″. The position is $23^h\ 42^m\ 33^s$, $-28°\ 41'{\cdot}7$.

7807. The position given by Stone is $23^h\ 56^m\ 33^s$, $-19°\ 19'{\cdot}6$. The position found by me is $23^h\ 55^m\ 19^s$, $-19°\ 23'{\cdot}8$. On one good night I looked carefully in the place given by Stone and saw no nebula.

The Bruce Micrometer of the Chamberlin Observatory.

By Herbert A. Howe.

(*Communicated by the Secretaries.*)

Through the liberality of Miss Catherine W. Bruce, of New York City, a micrometer adapted to observations of $\Delta\alpha$ and $\Delta\delta$ was built some eighteen months since for the Chamberlin Observatory, by Saegmuller, of Washington, D.C. It was especially intended for work on faint nebulæ. Each nebula is tied to some small star near at hand by micrometric measures of $\Delta\alpha$ and $\Delta\delta$. On some ensuing night, when the Moon obliterates most nebulæ, the comparison star is connected by chronographic and micrometric observations with a catalogued star. The instrument has also been used for a long series of observations of *Eros*, and for similar work on some faint comets. So excellent has been its performance, and so efficient is it for rapid work, that I have been asked by a visiting astronomer to write a detailed description of it.

1. Such a micrometer is usually screwed on to the tail-piece of the equatorial. Whenever it is taken off and replaced, a new value of the parallel, and hence of the zero of the position-circle, must be obtained. The Bruce micrometer slides off, so that when it is put back, the zero of the position-circle is the same as before. The slipping piece, on which the micrometer slides into place, is screwed on to the tail-piece in the usual way, and is fastened by three small steel screws, so that it will not work loose. When the micrometer has been slid into place it is secured by two thumb-screws. The plane in which the sliding takes place is perpendicular to the sight-line of the telescope, and the edges of the run-way are parallel to the declination axis of the equatorial.

2. The position-circle is 9 inches in diameter, and furnished with a system of solid and accurately adjustable stops by which the box can be turned precisely 90^c when micrometric measures of $\Delta\alpha$ and $\Delta\delta$ are being made, without reading the graduated circle. If it is desired at any time to have the micrometer box rotate freely, as in measures of double stars, the stops can be thrown back in a few seconds, and when needed again can be swung with equal facility into their former positions. If the box be turned till it is brought up against a stop several times in succession, and the position-circle be read each time, the readings

will not differ a minute of arc. The circle can be clamped at any angle and a tangent screw used.

3. The eye-piece is swiftly movable a good distance both in right ascension and in declination, so that exceptionally long spider webs can be used. The right ascension wires are eleven in number, the central wire being flanked by two sets of five wires each. In each set the equatorial intervals are 3, 4, 4, and 3 seconds. From the central wire to the nearest of each flanking set is 10 seconds. The entire breadth of this wire system is therefore 48 seconds. By this arrangement three deliberate bisections of an object can be made with a declination wire while it is passing through the field of view, in addition to ten chronographic observations of time. The mean of the times of the bisections in declination is practically the same as the mean of the chronographic times.

The declination wires are nine in number; they are symmetrically spaced; the intervals of the four wires on one side of the central wire are 3′, 2′, 5′, and 5′. The total angular space spanned by the declination wires is therefore 30′. The entire system of declination wires is movable 10′ each side of its central position by the micrometer screw. Thus differences of declination exceeding 30′ may be measured without turning the micrometer screw much. In the hundreds of observations which have so far been made between two objects, one of which was a catalogued star, very few measures of $\Delta\delta$ have exceeded 20′.

The box is provided with an index, so that it may be placed in the centre of its run at any time. So accurately have the spider webs been put in that when the box is in the centre of its run the point of intersection of the two central webs remains upon a star, while the box is being turned through 180° in position angle.

4. For rapid work a coarse micrometer screw, having only twenty threads to the inch, has been considered essential. The screw has three large and boldly divided heads, two of which are movable. Each of the movable heads is furnished with an efficient clamp, which brings no perceptible side-strain on the micrometer screw. If three declination bisections are to be made as a star drifts through the field of view, the two movable heads are first set so that their readings agree with that of the fixed head. When the first bisection has been made, the outer head is clamped. After the next bisection the second head is clamped. For the third and final bisection no clamping is necessary. When the star has passed out of sight the three readings of the micrometer heads are noted. Even when the difference of right ascension of two equatorial stars is only a minute, ten observations of $\Delta\alpha$ and three of $\Delta\delta$ can be made without undue haste. These involve twenty chronographic registrations and six bisections in declination. When the two objects are only a few seconds apart in right ascension, the declination observations are usually made with the driving clock running.

5. The illumination of each set of wires is effected by two electric lamps of 2 c.p. each, and of about 5 volts. The lamps are so well encased that no stray light bothers the observer. They ordinarily glow with only a fraction of 1 c.p. apiece, and their intensities are so regulated by two rheostats and two mechanical shutters that any required intensity of light may be thrown on each set of wires at any time. In observing a very faint comet or nebula, it is sometimes of advantage to be able to turn the illumination in declination off and on instantaneously, or to have the two sets of wires simultaneously lighted up with different intensities. The lamps are fed by a current from a little transformer, which can be adjusted to any voltage required by them. Through the primary coil of the transformer the regular house-current of 50 volts runs. A four-point switch on the micrometer enables the observer to throw the light on the two verniers of the position-circle or on either set of wires, or on both sets simultaneously.

On the Relation between Magnetic Disturbance and the Period of Solar Spot Frequency. By William Ellis, F.R.S., formerly of the Royal Observatory, Greenwich.

Having in previous papers * compared the variations of diurnal magnetic range with the period of solar spot frequency, I have since given some attention to the relation of magnetic disturbance to solar spot frequency, comparing, as before, results derived from the photographic magnetic records, for so many years maintained at the Royal Observatory, Greenwich, with the corresponding values of sun-spot frequency as tabulated by the late Dr. Rudolf Wolf, of Zürich. Before, however, describing what has been done in this respect, it may be well to make brief reference to some conclusions arrived at in the previous papers.

Magnetic variations, as registered from day to day, include principally the effects of two distinct phenomena : (1) the solar diurnal variation, a local phenomenon which, although everywhere existing, has distinct relation to the local day ; and (2) the apparently irregular magnetic disturbance, a phenomenon of more cosmical character, since when considerable, as in magnetic storms, it is felt simultaneously over widely separated districts of the Earth's surface, the commencement of disturbance when abrupt, as is frequently the case, being absolutely simultaneous at all places, so far as the contracted time scale used in photographic registers will allow us to determine. Magnetic disturbance is not always present ; some days are altogether free from magnetic irregularity, exhibiting only the ordinary diurnal movement, but on other days magnetic irregularity, superposed

* *Phil. Trans.* for 1880, p. 541, and *Proc. Royal Society*, vol. 63, p. 64.

on the diurnal movement, exists to a lesser or greater extent. In the papers referred to I have shown the close relation that exists between the variation of diurnal magnetic range and the period of solar spot frequency, the diurnal range during the years 1841 to 1896, as recorded at the Royal Observatory, Greenwich, being, both in declination and horizontal force, greatest when sun-spots are at a maximum, and least when sun-spots are at a minimum, the irregularities in both phenomena being distinctly similar, not only as regards variation in the length of successive periods (maximum to maximum or minimum to minimum), but also in the degree of intensity at the different epochs of maximum effect.

The diurnal range compared with sun-spot frequency during the period mentioned was the monthly mean diurnal range as found by employing all days of observation, including disturbed days (omitting only a few of extreme disturbance). It was, however, further shown that if quiet days are alone employed—that is, days free from magnetic irregularity - the resulting values of mean diurnal range were altogether similar, in that in both cases there was the same marked variation in value concurrent with the variation in sun-spot frequency.*

There are necessarily considerable differences in the magnitude of the diurnal range on adjacent days when disturbance is present, and the explanation of the comparatively small effect that the inclusion of days of such character produces in mean values appears to be that irregular disturbance acts sometimes to increase and sometimes to decrease the value of the magnetic element, these opposite effects tending in the aggregate to neutralise each other to such an extent as to influence the resulting diurnal range in a minor degree only as compared with the very considerable variation in value between the epochs of minimum and maximum of sun-spots. The circumstance that the variation of diurnal range with variation of sun-spot frequency exists in a similar marked degree whether days of disturbance are or are not included shows that the variation observed is essentially that of the undisturbed solar diurnal range. Otherwise the increased range observed at epochs of sun-spot maximum might be supposed to be in a measure due to the influence of the magnetic disturbance at such epochs prevalent.†

* The mean diurnal range of declination on days of quiet magnetism was for the year 1889 (near sun-spot minimum), 6′·45, and for 1893 (near sun-spot maximum) 10′·13. The corresponding values found by using all days were 6′·67 and 10′·40 respectively, the ratio between the two numbers being in both cases the same. Similarly also for horizontal force. See the *Report of the British Association Committee on Comparing and Reducing Magnetic Observations* for 1898, p. 106.

† To persons not having had the opportunity of examining a collected series of photographic magnetic registers it would probably create surprise to see how strongly marked is the difference in the character of the records at sun-spot maximum and sun-spot minimum; as, for instance, between the records for the years 1870 and 1879. Without measurement or calculation the difference is evident to mere inspection, not only in the abundance of

Having recapitulated so much as is necessary to show that the variation of diurnal range with variation of sun-spots appears to be one independent of the effect of magnetic disturbance, we can now proceed to consider what may be really the relation of magnetic disturbance itself to sun-spot frequency. Regular magnetic observations were commenced at Greenwich in 1841; but the records for the years 1841 to 1847, depending mainly on two-hourly eye observations, although sufficient for dealing with the question of diurnal range, do not give that continuous record necessary for the present purpose. But photographic registration was commenced in the year 1848, enabling us to employ here the records for the fifty years 1848 to 1897. I would here gratefully acknowledge the kindness of the Astronomer Royal in permitting me to use the Royal Observatory records in this inquiry, and would also add that, for any opinions herein expressed or conclusions drawn, I am alone responsible.

To make any complete or extensive discussion of magnetic disturbance through a long period would be an extremely laborious task. And my object being simply to compare the frequency of magnetic disturbance with sun-spot frequency, I have contented myself with an endeavour to bring out the main features of the frequency of different degrees of magnetic disturbance, and even what I have attempted has proved to be a considerable piece of work. But a similar presentation of the characteristics of disturbance in the direction mentioned has not, I think, been before made.

As a measure of the frequency of magnetic disturbance I have counted up the number of days on which during any part of the twenty-four hours (of the astronomical day, reckoning from noon), magnetic movement or disturbance of sudden or transient character, either in declination or horizontal force, reached a stated amount, dividing the days for this purpose into five classes, according to magnitude of disturbance. In this classification any day on which was recorded one irregularity coming within the stipulated limits, either in declination or horizontal force, was included, but usually there would be on the same day other similar irregularities. This system of selection, admittedly of arbitrary character, if open to criticism, nevertheless brings out some interesting distinguishing features in regard to the behaviour of magnetic disturbance. That the interpretation of the records should be similar was here more important than absolute accuracy of classification. In order, therefore, to secure, as far as possible, comparative results—since different persons might vary in their methods of applying a given rule of selection—the whole selection was made by myself, according to the annexed scheme. Class I. includes all days free from magnetic irregularity, showing only the ordinary diurnal movement:—

disturbance in 1870 and its paucity in 1879, but also in the sensibly greater boldness of the diurnal curve in the former year on days of quiet magnetism.

Class.	Degree of Disturbance.	Greatest recorded Magnetic Movement. Declination.	Horizontal Force. c.g.s. value × 10^5.
I.	None	No magnetic irregularity in either element.	
II.	Minor	Less than 10′	Less than 50.
III.	Moderate	Greater than 10′ less than 30′	Greater than 50. less than 150.
IV.	Active	Greater than 30′ less than 60′	Greater than 150. less than 300.
V.	Great	Greater than 60′	Greater than 300

60′ in declination is approximately equivalent in horizontal force to 300 (c.g.s. value × 10^5).

In Table I. there is given for each quarterly period during the years 1848 to 1897 the number of days of each degree of disturbance, according to the convention described. In dividing the year into quarterly periods defined by calendar months (January, February, and March being taken as the first quarter, and so on), a little inequality is introduced in dealing with sums instead of means, owing to the slightly different length of the several quarters which contain 90, 91, 92, and 92 days respectively (February 29 being for convenience altogether omitted, nothing unusual having occurred on this day in the different leap years included); but, remarking the very considerable variation in the successive numbers of the table, the slight inequality in the length of different quarters is immaterial. On occasions of defective or deficient photographic register at Greenwich reference was made to those of the Kew Observatory, with the kind permission of the Superintendent, the late Mr. G. M. Whipple. On some similar occasions in the earlier years the degree of disturbance was estimated from eye observations.

TABLE I.

Number of Days of Magnetic Disturbance of Different Degrees of Intensity in Quarterly Periods during the Years 1848 *to* 1897, *as deduced from the Records of the Royal Observatory, Greenwich.*

(The quarters of the year, indicated by the figures 1 to 4, are taken in the ordinary way, the first quarter comprising the months of January, February, and March, and so on.)

Year.		Degree of Disturbance. None.	Minor.	Moderate.	Active.	Great.	Year.		Degree of Disturbance. None.	Minor.	Moderate.	Active.	Great.
1848	1	9	50	24	6	1	1850	1	20	53	16	1	...
	2	16	51	22	2	...		2	8	69	14	...	...
	3	20	58	13	1	...		3	10	71	11	...	...
	4	7	54	23	6	2		4	23	61	8	...	...
1849	1	9	63	16	2	...	1851	1	16	58	13	3	...
	2	10	69	12	...	...		2	10	75	6	...	...
	3	7	63	22	...	...		3	7	65	14	4	2
	4	11	61	15	5	...		4	4	62	21	3	2

Year.		Degree of Disturbance. None.	Minor.	Moderate.	Active.	Great.	Year.		Degree of Disturbance. None.	Minor.	Moderate.	Active.	Great.
1852	1	9	43	30	5	3	1861	1	11	62	11	6	...
	2	4	68	15	4	...		2	7	76	8	...	...
	3	13	61	18	...	...		3	8	76	8	...	...
	4	10	57	23	1	1		4	15	60	15	2	...
1853	1	16	55	19	...	...	1862	1	13	65	10	2	...
	2	13	60	16	2	...		2	11	72	8	...	...
	3	17	64	8	2	1		3	2	58	26	4	2
	4	14	66	9	3	...		4	1	62	24	4	1
1854	1	5	44	35	4	2	1863	1	3	51	32	4	...
	2	10	60	20	1	...		2	6	71	14	...	...
	3	23	57	11	1	...		3	4	63	24	1	...
	4	29	54	8	1	...		4	12	56	23	1	...
1855	1	15	61	13	1	...	1864	1	13	56	20	1	...
	2	22	58	11	...	...		2	11	48	29	3	...
	3	13	71	8	...	...		3	4	55	31	2	...
	4	24	61	7	...	...		4	5	45	41	1	...
1856	1	17	64	9	...	...	1865	1	4	52	32	2	...
	2	32	56	3	...	...		2	11	65	14	1	...
	3	27	51	14	...	...		3	8	52	28	2	2
	4	28	53	11	...	...		4	11	54	24	3	...
1857	1	19	63	8	...	...	1866	1	5	58	22	4	1
	2	13	69	8	...	1		2	10	73	8	...	...
	3	3	80	8	1	...		3	8	71	13	...	...
	4	11	63	17	...	1		4	2	64	25	1	...
1858	1	8	52	26	4	...	1867	1	10	65	14	1	...
	2	4	59	25	2	1		2	9	72	10	...	...
	3	3	61	28	...	...		3	2	69	21	...	...
	4	4	57	29	2	...		4	7	68	17	...	...
1859	1	9	58	22	1	...	1868	1	6	64	20	...	...
	2	8	60	19	4	...		2	6	64	20	1	...
	3	6	63	16	3	4		3	6	63	19	3	1
	4	8	57	24	2	1		4	8	68	15	1	...
1860	1	5	64	18	3	...	1869	1	3	59	26	2	...
	2	2	71	14	4	...		2	...	62	23	4	2
	3	2	54	24	9	3		3	3	57	30	2	...
	4	13	67	12	...	...		4	9	63	20	...	...

Year.		Degree of Disturbance. None.	Minor.	Moderate.	Active.	Great.	Year.		Degree of Disturbance. None.	Minor.	Moderate	Active.	Great.
1870	1	4	50	33	2	1	1879	1	20	68	2	...	..
	2	4	53	31	2	1		2	39	50	2	...	...
	3	3	60	22	6	1		3	24	65	3	..	...
	4	8	63	16	3	2		4	28	62	2	...	...
1871	1	9	58	20	2	1	1880	1	39	45	6	...	...
	2	5	56	25	3	2		2	24	57	10	...	...
	3	2	64	23	2	1		3	15	60	15	2	...
	4	3	69	17	1	2		4	10	59	22	1	...
1872	1	4	54	31	...	1	1881	1	12	59	18	...	1
	2	2	67	18	4	...		2	21	61	9	...	...
	3	4	57	24	4	3		3	11	71	8	1	1
	4	2	55	29	4	2		4	4	63	25	...	...
1873	1	3	39	41	7	...	1882	1	10	62	17	1	...
	2	...	53	36	2	...		2	6	57	23	2	3
	3	1	54	37	...	...		3	5	66	19	2	...
	4	11	64	17	...	...		4	2	47	31	7	5
1874	1	7	62	17	3	1	1883	1	6	48	34	2	...
	2	6	66	16	3	...		2	3	66	20	2	...
	3	6	66	20	...	...		3	4	57	28	3	...
	4	14	55	21	1	1		4	14	61	14	3	...
1875	1	19	57	14	...	...	1884	1	16	61	13	...	...
	2	15	62	14	...	...		2	10	67	14	...	...
	3	17	68	7	...	...		3	13	67	9	3	...
	4	22	59	11	...	...		4	13	65	11	3	...
1876	1	22	59	8	1	...	1885	1	14	61	13	2	...
	2	23	67	1	...	...		2	7	67	13	4	...
	3	19	70	3	...	...		3	7	65	19	1	...
	4	23	57	12	...	...		4	19	63	10	...	...
1877	1	21	64	5	...	...	1886	1	13	52	20	4	1
	2	18	63	9	1	...		2	1	62	26	2	...
	3	26	58	8	...	...		3	...	68	23	1	...
	4	29	56	7	...	...		4	2	52	35	3	...
1878	1	26	62	2	...	...	1887	1	11	58	21	...	...
	2	33	54	3	1	...		2	5	70	16	...	...
	3	29	63	...	...	...		3	11	68	12	1	...
	4	31	58	3	...	...		4	13	65	13	1	..

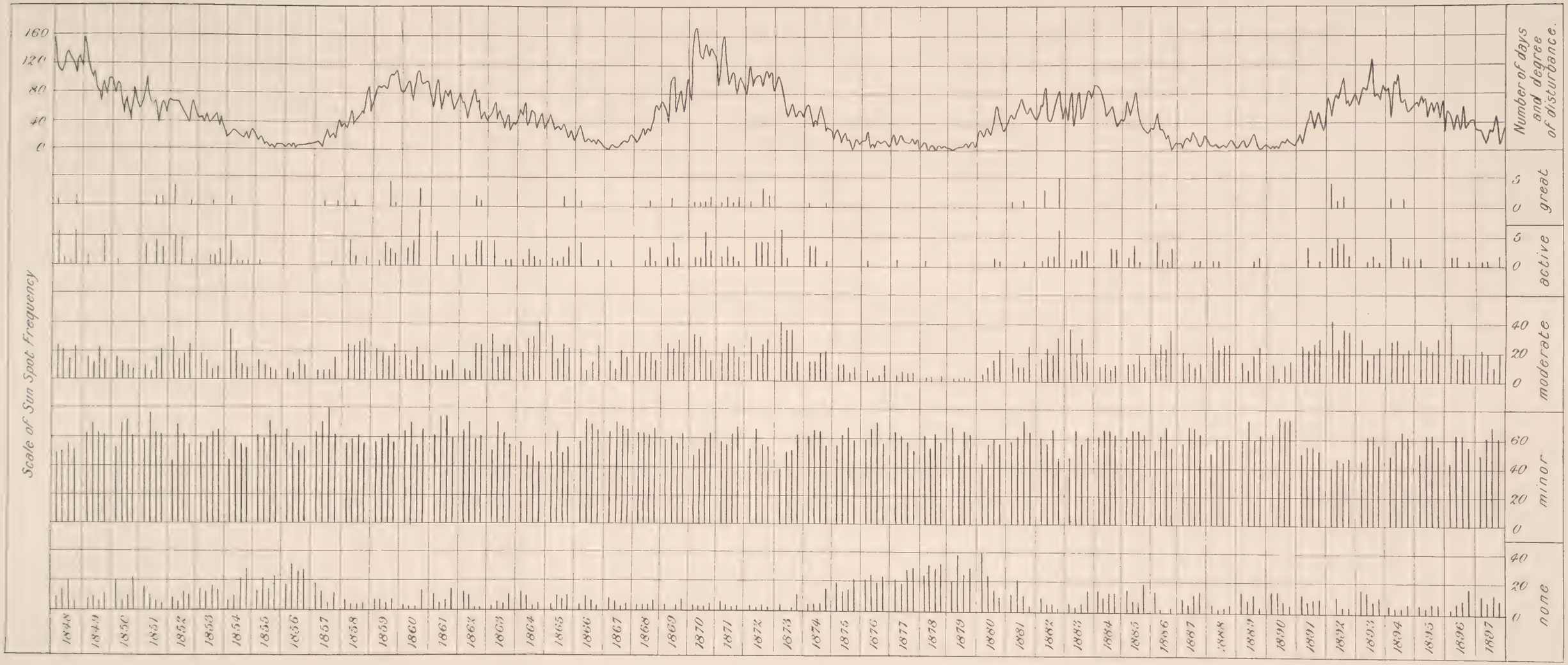
Year.		Degree of Disturbance. None.	Minor.	Moderate.	Active.	Great.	Year.		Degree of Disturbance. None.	Minor.	Moderate.	Active.	Great.
1888	1	7	51	31	1	...	1893	1	14	46	30	...	...
	2	3	61	26	1	...		2	13	62	15	1	...
	3	4	60	28	...	...		3	7	63	20	2	...
	4	5	60	27	...	...		4	10	57	24	1	...
1889	1	14	60	16	...	...	1894	1	6	50	27	5	2
	2	8	74	9	...	...		2	3	60	28	...	...
	3	12	60	19	1	...		3	3	65	20	2	2
	4	3	62	25	2	...		4	7	61	23	1	...
1890	1	14	63	13	...	...	1895	1	7	52	30	1	...
	2	14	75	2	...	...		2	3	64	24	...	...
	3	9	72	11	...	...		3	7	63	22	...	...
	4	6	73	13	...	...		4	7	55	30	...	...
1891	1	13	52	25	...	...	1896	1	2	44	42	2	...
	2	8	57	23	3	...		2	7	65	17	2	...
	3	9	56	27	...	...		3	9	64	19	...	...
	4	9	52	30	1	...		4	18	57	16	1	...
1892	1	...	41	42	3	4	1897	1	15	51	23	1	...
	2	11	50	24	5	1		2	9	61	20	1	...
	3	4	45	37	4	2		3	13	68	11	...	...
	4	5	49	36	2	...		4	9	61	20	2	...

The division into quarters adopted in this table tends to elimination of the influence of the seasonal inequality that, according to Table II., appears to exist.

The numbers of Table I., indicating in each class the frequency of a given degree of disturbance, are represented graphically on Plate 4. The scale of days is of equal length on the paper for the groups of no disturbance, and those of minor and moderate disturbance, but is quadrupled in length for the groups of active and great disturbance; since the number of days in these two divisions being comparatively few, the variation in number would not otherwise have been readily perceived. Above these representations of different degrees of magnetic disturbance, there is added the corresponding curve of sun-spot frequency plotted from Wolf's monthly values, derived directly from observation, as given in his *Astronomische Mittheilungen*, No. 50, and other succeeding numbers.

Examining the plate, we see at once that unusual magnetic disturbance is frequent about epochs of sun-spot maximum, and nearly or quite absent about epochs of sun-spot minimum, the general rise and fall in the number of days of great and active disturbance, concurrent with variation in sun-spot frequency, being sufficiently evident. In the approach towards sun-spot

Curve of observed values (monthly means) of Sun Spot Frequency (Wolf); and number of days of Magnetic Disturbance of different degrees of intensity, in quarterly periods, during the years 1848 to 1897, as deduced from the records of the Royal Observatory, Greenwich.



minimum there comes usually a time at which great disturbance ceases, whilst instances of active disturbance continue to occur; these ultimately also cease for a shorter or longer period at sun-spot minimum, after passing which disturbance is quickly renewed with rapidly increasing magnetic activity, in striking contrast to the more gradual dying down of activity as sun-spot mimimum is approached. Disturbance classed as great, including those seemingly irregular outbursts of the first magnitude when the magnets are so violently agitated, although more closely confined to the periods immediately preceding and immediately following epochs of sun-spot maximum, sometimes appears a considerable time afterwards. Thus there are instances of great disturbance between 1851 and 1854, following the sun-spot maximum of 1848, and in 1865 and 1866 following the sun-spot maximum of 1860; and there is a recent instance in 1898 following the sun-spot maximum of 1894, when in the years 1895 to 1897 no disturbance of the class great having occurred, there were two in 1898, on March 15 and September 9 respectively. Moderate disturbance is diminished in a remarkable degree at the sun-spot minimum of 1879, and in a lesser degree at that of 1856; at other of the epochs of sun-spot minimum there is little variation. In the class of minor disturbance there seems to be no definite variation concurrent with that of sun-spots. But the days of no disturbance show distinct increase in number at the sun-spot minimum of 1856, and a marked increase at the minimum of 1879. The cessation of great disturbance during the years 1875 to 1880, the small number of days of active disturbance, the diminished number of those of moderate disturbance, and the greatly increased number of those of no disturbance show the period about the sun-spot minimum of 1879 to have been one of extraordinary magnetic quiet, following as it did the unusually active sun-spot maximum of 1870.

The intervals at the several epochs of sun-spot minimum without magnetic disturbance of the class great were as follows :—

Period free from great disturbance.	Interval.	Epoch of sun-spot minimum.	Interval before minimum.	Interval after minimum.
1854·2 to 1857·3	3·1	1856·0	1·8	1·3
1866·1 to 1868·7	2·6	1867·2	1·1	1·5
1874·8 to 1881·1	6·3	1879·0	4·2	2·1
1886·2 to 1892·2	6·0	1890·2	4·0	2·0

The dead time at sun-spot minimum as regards great disturbance is thus variable in length, and the actual sun-spot minimum occurs when the interval is long towards the end of the dead time; but after the minimum is passed magnetic activity soon arises, more tardily, however, when the whole interval is long, probably consequent, as an old sun-spot cycle dies out, on the more or less speedy commencement in high solar latitudes of a

new cycle. In all, during the fifty years discussed, there were thus in the aggregate 18·0 years without any disturbance of degree here classed as great.

It may be interesting to add a list of the actual days on which, at Greenwich, disturbance of unusual character occurred. The list includes all days of the class of great disturbance, and a certain number of those of the class of active disturbance that approach the upper limit of this class. An asterisk affixed to the date indicates that the disturbance was of remarkable character. Occasionally disturbance would arise on the day next preceding, or would continue to the day next following, but more usually it was confined to the day mentioned. In cases, however, in which there was any accompanying prolonged disturbance, the interval over which it extended is added in brackets.

1848 January 16, February 21 (20 to 24), July 11, October 18,* November 17,* December 17.
1851 February 18, September 3, 6, 7* (3 to 7), September 29,* October 2 (September 29 to October 2), December 6, 29.
1852 February 14, 15, 17,* 18, 19* (14 to 21), June 11, November 11.*
1853 May 24, July 12.
1854 February 24, March 15, April 10.
1857 May 7, December 16.*
1858 April 9.*
1859 April 21, August 28,* September 1,* 2*, 3 *(August 28 to September 5), October 12.*
1860 August 8, 9, 10, 12* (6 to 12), September 6.
1861 January 24, 25, 26 (22 to 27), December 19.
1862 July 5, August 4, October 3* (3 to 6), December 14.
1864 July 19.
1865 August 2, 3, 4 (2 to 5).
1866 February 20, 21, 25 (20 to 25).
1868 September 30.
1869 February 3, April 15,* May 13,* September 13.
1870 February 1, April 5, May 20,* August 20, September 24* (23 to 26), October 24,* 25.*
1871 February 12, April 1, 9, June 17,* August 24, November 2,* 10.
1872 February 4,* April 10, July 7,* 8,* August 3, 4, 8, 14, October 14,* 15, 16, 17* (14 to 18).
1873 January 7 (3 to 7), March 9.
1874 February 4, October 3 (3 to 5).
1880 August 12, 13.
1881 January 31*, September 12 (12 to 14).
1882 April 16,* 17,* 19* (16 to 20), June 24, August 4, October 2, November 16,* 17,* 19,* 20,* 21 (12 to 21).
1883 February 24, September 15, 16.
1884 July 3, November 2.
1885 March 15.

1886 January 9, March 29,* 30.
1892 February 13,* March 1, 6, 11, 12* (1 to 12), April 25, 26, May 18,* June 26, July 16,* August 12.*
1894 February 23, 25, 28 (20 to 28), March 20, July 20,* August 19,* September 14.

Some of the most remarkable disturbances were those of 1848 November 17, 1859 August 28 to September 5, 1872 February 4 and October 14 to 18, 1882 April 16 to 20 and November 12 to 21 (accompanied on November 17 by a very remarkable aurora), and 1892 February 13.

In cases of prolonged disturbance there is frequently a second outbreak after an intervening interval of moderate disturbance, and even of comparative quiet. Instances of this occurred in the disturbance of 1851 September 3 to 7, and that of September 29 to October 2 of the same year, also in those of 1852 February 14 to 21, 1859 August 28 to September 5, and 1882 April 16 to 20. In the disturbed period of 1892 March 1 to 12 there were three outbursts with intermediate moderate disturbance. On the other hand in the disturbance of 1860 August 6 to 12 considerable activity was maintained throughout, also in that of 1872 October 14 to 18, and in other cases. Remarkable disturbances occurred during a quiet period on 1870 October 24 and 25, accompanied on both nights by a fine aurora. Indeed, aurora was visible on many of the dates of the preceding list, but it formed no part of my intention to consider on the present occasion the relation of the aurora to magnetic disturbance.

We have seen that the magnetic activity prevalent about epochs of sun-spot maximum becomes greatly subdued or disappears about epochs of sun-spot minimum, indicating general relation between the two phenomena; and the appearance of a very large solar spot is usually accompanied by considerable magnetic disturbance, existing it may be for many days, as in 1882 November. But it is difficult to establish correspondence in details, or connect definite solar change with definite magnetic movement. The comparison is indeed one not easy to make, for although we possess continuous registers of magnetic variations no continuous record of solar change seems possible, and is indeed impossible for the face of the Sun that is turned away from the Earth. It is known that magnetic disturbance will break out with great suddenness simultaneously at widely different parts of the Earth's surface, indicating influence of cosmical character*; but the question is what action, solar or otherwise, determines the moment when simultaneously the whole Earth shall feel the initial magnetic impulse. Such sudden movements are not at all infrequent, and commonly presage magnetic disturbance or magnetic storm. I once began to make a list of the sudden initial movements that are to be found in the Greenwich records, and I believe it would be of interest to complete the list,

* *Proc. Royal Society*, vol. 52, p. 191.

as it appeared to me that there was some reason to suppose that the movements in question were more numerous when the Earth in its diurnal rotation stood in a given position in relation to the Sun. There is a further difficulty in regard to the comparison of solar and magnetic phenomena. Solar change and magnetic disturbance may either of them at times occur in marked degree without any apparently corresponding magnetic effect answering to the solar, or solar answering to the magnetic, or indeed any correspondence that can be distinctly traced. Thus the facts of observation, so far as known, are themselves in a degree perplexing.

Having dealt with the behaviour of magnetic disturbance from year to year, the same material may now be employed to consider more closely the variation in the frequency of disturbance through the course of the year; that is, develop from the aggregate of years 1848 to 1897 the character of the annual inequality as regards disturbance. Using the same convention as before for indicating different degrees of disturbance, there is given in Table II. the number of days of no disturbance, and of minor, moderate, active, and great disturbance, for half-monthly periods on the aggregate of years mentioned. In dividing 365 days into twenty-four parts, omitting as before February 29, there arise nineteen portions of fifteen days each and five portions of sixteen days each. The difference is one too large here to neglect. The numbers in each 16-day period have therefore been reduced by one-sixteenth part to make the whole comparative, and for these periods it is the corrected numbers that are given in Table II. It will be observed that this table contains no sun-spot values, the variation of solar-spot frequency having no annual period. It may, however, be desirable before proceeding further to show from the sun-spot numbers that this is the case. If there be an annual inequality, monthly values taken through a number of years should tend, as the aggregate of years employed is increased, to elimination of the eleven-year period, whilst at the same time bringing out an annual period, if any definite annual period exists. Having regard to the seasonal character of the variation to which the numbers of Table II. are subject, it will be sufficient to tabulate the sun-spot values for quarterly periods. Forming such quarterly values from Wolf's numbers, those derived directly from observation, combining together the fifty years 1848 to 1897, and taking the months of February, March, and April to represent Spring, and May, June, and July, to represent summer, and so on, we have :—

1848–1897 (50 years).	Spring.	Summer.	Autumn.	Winter.	Mean.
Mean sun spot number	50·5	49·3	48·3	46·7	48·7
					Numerical Sum.
Differences from Mean	+ 1·8	+ 0·6	− 0·4	− 2·0	4·8

As the separate quarterly sun-spot values, those of individual years, vary from 0 at sun spot minimum to 150 or more at sun-

spot maximum, these residuals are insignificant. But further, since Wolf has given monthly sun-spot values commencing with 1749, quarterly means can be formed in the same way as before for the whole period 1749 to 1897 : these are 47·4, 47·8, 47·5, and 47·0, respectively. Mean, 47·4. Differences from mean, 0·0, +0·4, +0·1, and −0·4. Numerical sum, 0·9, from a series of 149 years, as compared with 4·8 from a series of 50 years, an approach to equality of value, as the number of years employed is increased, that entirely negatives the supposition of the existence of an annual period.

TABLE II.

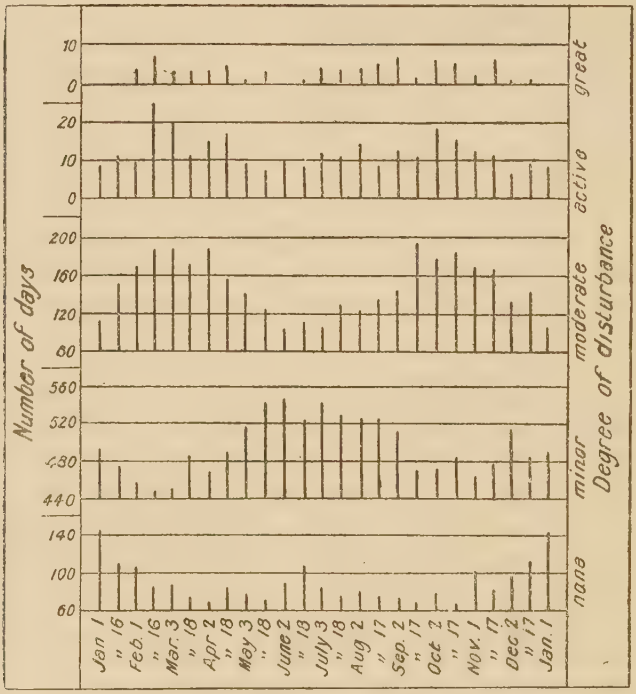
Number of Days of Magnetic Disturbance of Different Degrees of Intensity in Half-monthly Periods on the Aggregate of the Years 1848 *to* 1897, *as deduced from the Records of the Royal Observatory, Greenwich.*

Half Monthly Period.		Number of Days.	Middle Day of Period.	Degree of Disturbance. None.	Minor.	Moderate.	Active.	Great
Dec. 25	Jan. 8	15	Jan. 1	141	493	108	8	—
Jan. 9	23	15	16	108	477	154	11	—
24	Feb. 8	16	Feb. 1	106	460	171	9	4
Feb. 9	23	15	16	83	448	187	25	7
24	Mar. 10	15	Mar. 3	87	454	188	19	3
Mar. 11	25	15	18	78	483	174	11	3
26	Apr. 9	15	Apr. 2	70	472	189	15	4
Apr. 10	25	16	18	85	487	157	17	4
26	May 10	15	May 3	78	517	145	9	1
May 11	25	15	18	73	545	122	7	3
26	June 9	15	June 2	89	547	104	10	—
June 10	25	16	18	108	523	110	8	1
26	July 10	15	July 3	85	544	106	12	3
July 11	25	15	18	80	525	131	11	3
26	Aug. 9	15	Aug. 2	85	525	122	14	4
Aug. 10	24	15	17	77	524	136	8	5
25	Sept. 9	16	Sept. 2	74	514	143	13	6
Sept. 10	24	15	17	72	469	195	12	2
25	Oct. 9	15	Oct. 2	79	469	178	18	6
Oct. 10	24	15	17	66	481	182	16	5
25	Nov. 8	15	Nov. 1	101	465	169	13	2
Nov. 9	24	16	17	86	479	166	12	7
25	Dec. 9	15	Dec. 2	97	514	131	7	1
Dec. 10	24	15	17	111	483	146	9	1

From the numbers of Table II. the annexed diagram has been constructed. For the same reason as before the scale of days is made equal in length on the paper for the groups of no disturb-

ance and those of minor and moderate disturbance, but is quadrupled in length for the groups of active and great disturbance. We see in the diagram that great, active, and moderate disturbance is more frequent in spring and autumn than in summer and winter, the days of quiet magnetism, as represented by the classes of no disturbance and of minor disturbance, being correspondingly diminished at these seasons of the year. Conversely, when in summer and winter the frequency of great, active, and moderate disturbance declines, the days of quieter

Number of Days of Magnetic Disturbance of Different Degrees of Intensity in Half-Monthly Periods on the Aggregate of the years 1848 *to* 1897, *as deduced from the Records of the Royal Observatory, Greenwich.*



The day is the middle day of each half-monthly period.

magnetism become correspondingly increased—that is, magnetic activity at these seasons sinks to a lower level. It should be observed that the diagram represents the seasonal variation resulting from the combination of the whole fifty years 1848 to 1897, in which period years of different degrees of magnetic intensity occur. But, alike in years of lessened activity as well as in the more active years, the seasonal variation in the several classes of disturbance shows the same type of a diminished activity in summer and winter as is seen in the diagram.

The circumstance that magnetic activity is subject during the year to two maxima at or near the equinoxes, and two minima at or near the solstices, has been remarked at other places, M. Moureaux * having found a similar result from the discussion of the records of declination and horizontal force during the five years 1883 to 1887 at Parc Saint-Maur, as did Sabine † also previously from observations of declination at Toronto during the three years 1843 to 1845, the principal maximum in both cases falling at the autumnal equinox. In the Greenwich result there is no very marked difference between the spring and autumn maxima, and not very much difference between the summer and winter minima, although the winter seems to be the more quiet time. From observations made at Hobarton, during the same three years as at Toronto, Sabine found only one progression in the year, with a maximum in winter and a minimum in summer. Hobarton being in nearly the same latitude south as Toronto is north, this result would seem to require confirmation. It is, however, interesting that there should be such accordance as is above shown, considering that the Greenwich records have been treated in a somewhat different manner from that employed by Moureaux and Sabine. It is to be noted also that the Greenwich results depend on observations continued through a period of fifty years.

Magnetic disturbance is, as we see, most active around epochs of sun-spot maximum, and almost disappears about epochs of sun-spot minimum, both phenomena proceeding concurrently through a period of about eleven years. But by grouping together the same calendar month of different years it appears that magnetic activity is also greater in spring and autumn than in summer and winter, the seasonal difference in this respect being strongly marked. Thus, whilst sun-spot frequency continuously progresses from maximum to minimum, and from minimum again to maximum, there is superposed on the corresponding progression of magnetic disturbance an annual variation, having maxima at the equinoxes and minima at the solstices, to which, as has been shown in this paper, there is no counterpart in the progression of sun-spot frequency. It would thus appear that the progression in the case of magnetic disturbance becomes modified by the influence of terrestrial action. At the equinoxes, when magnetic activity becomes increased, the Earth in its diurnal revolution presents successively its whole surface to the Sun, but at the solstices, when magnetic activity becomes diminished, the region about one or other terrestrial pole is continually in shadow. Here then is a succession of phases in the Earth's annual movement relatively to the Sun apparently harmonising in time with the seasonal variations of magnetic activity. It might be interesting to be able to compare these results with others

* *Annales du Bureau Central Météorologique de France*, 1887, *Mémoires*, p. B. 35.

† *Phil. Trans. for* 1851, p. 123.

obtained from the records of some station in the southern hemisphere, but no lengthened series of results for any southern station appears to be available. The circumstance that the diurnal magnetic range in our latitude is greatest in summer and least in winter, and in corresponding southern latitudes is, in like manner, greatest and least in the southern summer and winter respectively, indicates that similar astronomical circumstances are accompanied by similar magnetic conditions. Now in the frequency of magnetic disturbance there is, in our latitude, a double progression with maxima at the equinoxes and minima at the solstices. If here also similar astronomical circumstances are accompanied by similar magnetic conditions, we should expect to find in corresponding southern positions also maxima of disturbance frequency at the equinoxes and minima at the solstices, a point on which it would be interesting to have further information.

The two papers contained, one in the *Philosophical Transactions for the year* 1880, and the other in vol. 63 of the *Proceedings of the Royal Society* (1898), and the present paper, together complete the work that I originally contemplated doing in regard to sympathetic relation between terrestrial magnetism and sun-spots. It may therefore be useful to add a brief summary of the results arrived at :—

1. Magnetic registers include mainly the effects of two phenomena, the solar diurnal variation and the irregular magnetic disturbance. On days of quiet magnetism the solar diurnal variation is alone present ; but on most days magnetic disturbance, superposed on the diurnal movement, exists to a lesser or greater extent.

2. The diurnal range of declination and horizontal force at the Royal Observatory, Greenwich—in forming which days of magnetic disturbance are included, excepting only those of extreme character—is found to vary, both in period and magnitude, in close accordance with the variation of sun-spot frequency throughout the series of years employed.

3. It was afterwards found that this variation is of the same character in every respect when days of quiet magnetism are alone employed ; the variation is consequently a feature of the undisturbed diurnal range.

4. A distinct relation also exists between magnetic disturbance, considered apart from diurnal variation, and sun-spot frequency, more or less marked, much more marked in some sun-spot periods than in others, there having been great magnetic activity at the sun-spot maximum of 1870, and unusual magnetic quiet at the sun-spot minima of 1856 and 1879—in a remarkable degree at the latter epoch.

5. That is to say, when the phenomena of diurnal variation and magnetic disturbance—which exist in combination

in the magnetic registers—are separated, it is found that each one in itself varies independently with the variation of sun-spot frequency.

6. Further, there is an annual inequality in the frequency of magnetic disturbance, having maxima at the equinoxes and minima at the solstices, to which there is no counterpart in the variation of sun-spot frequency.

Mean Areas and Heliographic Latitudes of Sun-spots in the year 1898, deduced from Photographs taken at the Royal Observatory, Greenwich, at Dehra Dûn (India), and in Mauritius.

(*Communicated by the Astronomer Royal*).

The results here given are in continuation of those printed in the *Monthly Notices*, vol. lix. p. 4, and are deduced from the measurements of solar photographs taken at the Royal Observatory, Greenwich; at Dehra Dûn, India; and at the Royal Alfred Observatory, Mauritius.

Table I. gives the mean daily areas of umbræ, whole spots, and faculæ for each synodic rotation of the Sun in 1898; and Table II. gives the same particulars for the entire year 1898 and the nine preceding years, for the sake of comparison. The areas are given in two forms: first, projected areas—that is to say, as seen and measured on the photographs, these being expressed in millionths of the Sun's apparent disc; and next, areas as corrected for foreshortening, the areas in this case being expressed in millionths of the Sun's visible hemisphere.

Table III. exhibits for each rotation in 1898 the mean daily area of whole spots, the mean heliographic latitude of the spotted area, and the mean distance from the equator of all spots; and Table IV. gives the same information for the year as a whole, similar results from 1889 to 1897 being added, as in the case of Table II. Tables II. and IV. are thus in continuation of the similar tables for the years 1874 to 1888, on pp. 381 and 382 of vol. xlix. of the *Monthly Notices*.

The rotations in Table I. and Table III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853, November 9. The assumed prime meridian is that which passed through the ascending node at mean noon on 1854, January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days. The dates of the commencement of the rotations are given in Greenwich civil time, reckoning from mean midnight.

TABLE I.

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Mean of Daily Areas. Projected. Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening. Umbræ.	Whole Spots.	Faculæ.
	d							
592	1897 Dec. 28·64	27	103	558	986	74	422	1136
593	1898 Jan. 24·98	28	142	799	969	92	528	1190
594	Feb. 21·32	27	211	1127	1045	147	781	1231
595	Mar. 20·64	27	36	196	755	27	150	856
596	Apr. 16·92	27	47	278	728	39	229	913
597	May 14·16	28	25	159	677	18	115	832
598	June 10·36	27	13	80	562	8	51	666
599	July 7·56	27	38	219	474	28	161	530
600	Aug. 3·77	26	121	657	546	79	437	670
601	Aug. 31·01	27	192	1194	857	134	863	984
602	Sept. 27·28	28	118	732	894	82	553	1007
603	Oct. 24·57	27	142	826	1007	88	516	1077
604	Nov. 20·87	26	62	336	682	48	247	740

TABLE II.

Year.	No. of Days on which Photographs were taken.	Mean of Daily Areas.					
		Projected			Corrected for Foreshortening.		
		Umbræ.	Whole Spots.	Faculæ.	Umbræ.	Whole Spots.	Faculæ.
1889	360	17·9	103	107	13·1	78·0	131
1890	361	21·3	133	273	15·5	99·4	304
1891	363	120	745	1322	86·2	569	1412
1892	362	255	1596	3230	186	1214	3270
1893	362	327	1983	2287	234	1464	2404
1894	364	317	1728	1666	231	1282	1877
1895	364	237	1330	2059	169	974	2278
1896	364	127	745	1243	90	543	1410
1897	364	122	695	977	88	514	1149
1898	363	93	532	767	64	375	891

TABLE III.

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Spots North of the Equator. Mean of Daily Areas.	Spots North of the Equator. Mean Heliographic Latitude.	Spots South of the Equator. Mean of Daily Areas.	Spots South of the Equator. Mean Heliographic Latitude.	Mean Heliographic Latitude of Entire Spotted Area.	Mean Distance from Equator of all Spots.
	d			°		°	°	°
592	1897 Dec. 28·64	27	281	8·61	142	9·60	+ 2·50	8·94
593	1898 Jan. 24·98	28	120	4·72	408	7·70	− 4·86	7·00
594	Feb. 21·32	27	125	10 50	656	11·45	− 7·92	11·30
595	Mar. 20·64	27	0	...	150	11·61	−11·61	11·61
596	Apr. 16·92	27	9	12·68	220	8·59	− 7·76	8·75
597	May 14·16	28	9	9·06	106	12·22	−10·49	11·96
598	June 10·36	27	19	15·13	33	10·06	− 0·85	11·92
599	July 7·56	27	76	10·29	85	12·41	− 1·69	11·41
600	Aug. 3·77	26	194	10·18	243	11·92	− 2·12	11·14
601	Aug. 31·01	27	68	8·25	795	11·69	−10·13	11·42
602	Sept. 27·28	28	164	13·53	389	10·88	− 3·65	11·67
603	Oct. 24·57	27	432	10·32	84	10·84	+ 6·88	10·41
604	Nov. 20·87	26	6	6·19	242	10·92	−10·53	10·81

TABLE IV.

Year.	No. of Days on which Photographs were taken.	Spots North of the Equator. Mean of Daily Areas.	Spots North of the Equator. Mean Heliographic Latitude.	Spots South of the Equator. Mean of Daily Areas.	Spots South of the Equator. Mean Heliographic Latitude.	Mean Heliographic Latitude of Entire Spotted Area.	Mean Distance from Equator of all Spots.
1889	360	5·0	7°·26	73·0	11°·90	−10°·68	11°·61
1890	361	53·1	22·20	46·3	21·75	+ 1·73	21·99
1891	363	401	20·49	169	19·91	+ 8·52	20·31
1892	362	607	15·09	607	21·69	− 3·29	18·39
1893	360	517	14·91	941	14·26	− 3·93	14·49
1894	364	543	12·31	739	15·56	− 3·75	14·18
1895	364	565	14·26	409	12·54	+ 3·01	13·54
1896	364	203	13·60	340	14·77	− 4·15	14·33
1897	364	196	8·32	318	7·73	− 1·62	7·96
1898	363	110	9·82	266	10·77	− 4·75	10·49

The principal features of the record for 1898 are :—

1. The decline in area of umbræ and whole spots, which seemed to have suffered a check in 1897, had resumed its regular course. The decrease in mean daily spotted area amounted to 27 per cent. for 1898 as compared with 1897, and to 31 per cent. as compared with 1896.

2. The decrease in the area of the umbræ has been in almost exactly the same proportion as for the whole spots—27 per cent. as compared with 1897, 29 per cent. as compared with 1896.

3. The decrease in the area of the faculæ has also been considerable—22 per cent. as compared with 1897, 37 per cent. as compared with 1896.

4. The decline in the whole spots has been chiefly in the northern hemisphere, the decrease as compared with 1897 being 44 per cent. for the northern hemisphere, but only 16 for the southern.

5. The predominance in spot activity of the southern hemisphere, noted in 1897, has become more striking in 1898.

6. The year has been marked by three chief outbreaks of spots. The first began on March 6, with the simultaneous appearance, at equal distances from the equator, of two fine groups in the same longitude, but one north and the other south of the equator. The greatest group of the year made its first appearance as one or two very small faint spots on August 11, and was quite insignificant up to the time of its disappearance at the W. limb after August 16. It had become a magnificent group by its return on September 3, and was still rapidly increasing. It attained its greatest area, 2,235 millionths of the visible hemisphere, on September 10, and then began to decline. It was still a fine group at its third appearance on September 30, but was rapidly diminishing, and at its fourth return on October 28 only a few small spots remained. The third chief group of the year was first seen on October 22, but appeared only for a single passage.

7. The chief characteristic of the year 1898 has been the return to a higher mean distance of the spots from the equator—viz., to 10°·5 instead of 8° in 1897. As this latitude accords well with that usually occupied by the spots at this stage of the decline towards minimum, 1897 stands out as having been quite abnormal, both as to its slight decline in area and its great decline in latitude.

8. The number of days without spots has increased, being 48 for 1898, as against 32 in 1897 and 8 in 1896. The number of days without faculæ was 11.

9. The year 1898 closely resembles the year 1886 as to mean daily spotted area, mean distance of spots from the equator, and number of days without spots. If the decline follows the course of the last cycle the next minimum should fall towards the end of 1901.

Observations of the Leonid Meteors of 1899 *made at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

Twenty meteors were observed in the morning of November 16, two of these during a partial break between 4.30 and 4.40 A.M., and the other eighteen during a clear period between 5.30 and 6.15 A.M. Thirteen of these meteors conformed to the *Leo* radiant. A continuous watch by three observers was kept from 11 P.M. to 6 A.M. on November 14–15 and November 15–16, but dense fog on the former night and cloud on the latter prevented any observations except during the above-mentioned periods.

Arrangements were also made to photograph meteors on the two nights, but owing to the weather no results were obtained.

A watch was also kept for the Bielid meteors on the nights of November 23 to 27, but the sky was generally covered with cloud, and no meteors were seen.

The following table shows the number of meteors seen at the Leonid epoch in each of the years 1887 to 1899 at Greenwich :—

Year.	Date of Observation.	Length of effective Watch.	Total Number of Meteors.	Number of Leonids.	Remarks.	No. of Observers.
1887	Nov. 15	A few minutes	2	1	Cloudy	1
1888	...	...	...	...	Quite cloudy	...
1889	Nov. 12–13	2^h	7	0	Cloudy generally	1
1890	...	...	...	...	Quite cloudy	...
1891	Nov. 13–14	2^h	0	0	Generally clear; bright moonlight	1
1892	Nov. 12–13	$1\frac{1}{2}^h$	0	0	Thin cloud	1
	,, 14–15	Brief	1	0	Cloudy generally	1
1893	Nov. 12–13	3^h	21	4	...	1
	,, 13–14	$1\frac{1}{2}^h$	9	1	Watch suspended on account of cloud	2
1894	...	...	...	...	Cloudy throughout	...
1895	Nov. 12–13	4^h	30	5	...	2
	,, 13–14	2^h	19	7	Observations stopped by cloud	2
1896	Nov. 12–13	2^h	8	2	...	3
	,, 15	$1\frac{1}{2}^h$	10	5	...	1
1897	Nov. 14	2^h	14	8	...	3
1898	Nov. 15–16	Brief	1	1	Cloudy	1
1899	Nov. 16	1^h	20	13	Cloudy	3+

Royal Observatory, Greenwich :
1899 *December* 8.

Observations of the Leonids of 1899, *made at the University Observatory, Oxford.* By H. H. Turner, Savilian Professor.

Preparations were made to observe the expected shower, but came to nothing. Mr. Bellamy had about seven cameras ready and four in reserve, to photograph the trails in different parts of the sky, and exposed some plates on November 14, but with no result. Messrs. H. Mullis, B. Gray, and E. Gray kept a regular watch on the sky in turns of half an hour each from $10^h{\cdot}30$ to $18^h{\cdot}0$ on the nights of November 14 and 15 ; and about thirty members of the University passed these nights at the Observatory, some of them watching, others sleeping on the floor of the Lecture-room, so that they might be quickly roused and carry the news to different colleges if there was a sensational shower. But nothing out of the common was observed. The following notes of details may be recorded :—

November 13. Cloudy at midnight. From $15^h\ 45^m$ to $16^h\ 15^m$, G.M.T., 5 Leonids and 2 other meteors seen by Professor Turner.

Nov. 14. Cloudy from $10\frac{1}{2}^h$ to 14^h. Cleared between 14^h and 15^h, and became a splendid night. The Moon and bright stars seen close to horizon. Moon set at 17^h.

The following meteors were counted by Messrs. H. Mullis, B. Gray, and E. Gray :—

G.M.T. Nov. 14.	Observer.	Number Leonids.	Of Others.
h h			
15 to $15\frac{1}{2}$	H.M.	2	—
$15\frac{1}{2}$,, 16	B.G.	4	1
16 ,, $16\frac{1}{2}$	E.G.	3	2
$16\frac{1}{2}$,, 17	H.M.	8	—
17 to $17\frac{1}{2}$	B.G.	9	—
$17\frac{1}{2}$,, 18	E.G.	18	—
18 ,, $18\frac{1}{2}$	H.M.	3	—

These observers were watching the radiant and its neighbourhood. A few other meteors not included in above were observed by Mr. Bellamy, who was in charge of the cameras in the dome, Mr. H. Hilton, Fellow of Magdalen, and Councillor G. C. Druce, who were with him, as follows :—

Sid. Time (Greenwich). h	m	s	
6	48	40	3rd mag. midway ζ and μ Leonis towards 20 Leonis path 12° long, rapid.
8	11	10	2° north of μ Leonis and R.A. 10^h.
8	12	25	East to west through Orion, long trail.
8	16	0	Near Procyon to Sirius, long trail.
8	25	10	In Leo.
8	56	10	Large meteor though Ursæ Min.
9	16	35	Near the zenith, the brightest seen during the night; caused a sudden glare of light over the landscape and left a trail for some minutes.

About 6 others were seen in *Orion*, and quite 25 near *Leo*, but times or paths were not noted, most of them until 18^h would be included in the above counts.

November 15. Cloudy all night; rifts in clouds 17^h to 18^h, but no meteors seen through them.

November 16. 11^h to 18^h. Brilliantly clear at times, but no meteors seen by Mr. Bellamy, who kept an intermittent watch.

Observations of the Leonids of 1899 November 14 and 15, made at the Radcliffe Observatory, Oxford.

(*Communicated by A. A. Rambaut, M.A., D.Sc., Radcliffe Observer.*)

At the Radcliffe Observatory we made careful preparations for observing the *Leonid* shower this year.

Our attention was chiefly devoted to an attempt to photograph some of the meteor trails and to an enumeration of the meteors seen. The staff of the Observatory being limited in number, it seemed to me best to confine observations to these two objects; and as the results obtained by Professor Pickering and one or two others in recent years seem to show that even a few photographic trails will give a better determination of the actual position of the radiant point than a much larger number charted down by eye and hand, I decided, after some hesitation, not to attempt any observations by the latter method.

For the photographic observations a camera carrying a Ross portrait lens of $4\frac{1}{2}$-inch aperture and about 18 inches focal length was attached to the declination axis of the Barclay 10-inch equatorial, the latter being used as a guiding telescope during the exposure of the plates. The camera was adapted for whole plates, which covered a field of about 30° by 20° on the sky.

From some experiments made previously with this apparatus by sweeping the telescope, we had concluded that if a meteor as bright as a first-magnitude star appeared in the neighbourhood of the radiant point during the exposure of a plate, we should be likely to obtain a photograph of its trail. The plates used were Edwards' Isochromatic Snap Shot Plates.

Unfortunately, however, although Mr. McClellan and I kept exposing plates from shortly after 15^h on the 14th until dawn put a stop to our observations, no trails were obtained.

Mr. Wickham and Mr. Robinson undertook the count of the meteors. They were stationed on the roof of the eastern wing of the Observatory tower. A cable had been temporarily laid to the chronograph, by which Mr. Wickham was enabled to register the time of appearance of each meteor to a fraction of a second. Mr. Robinson was also furnished with a pocket enumerator registering up to 12,000. Unfortunately, the richness of the display fell so far short of anticipation that the value of these

arrangements for registering the numbers was not put to the test.

Our watch commenced at 10h on November 14. At 11.30, however, the sky became overcast, and remained so until about 14h 30m, when the clouds lifted towards the east.

The number of *Leonids* observed in each quarter of an hour is given below. The actual times of appearance of thirty-one meteors observed by Mr. Wickham have been read to the nearest second from the chronograph, but it does not appear necessary to print these times.

G.M.T. h m	No. of Meteors Observed.	G.M.T. h m	No. of Meteors Observed.
14 45		16 30	
	5		3
15 0		16 45	
	2		5
15 15		17 0	
	2		4
15 30		17 15	
	3		8
15 45		17 30	
	4		9
16 0		17 45	
	0		5
16 15		18 0	
	3		6
16 30		18 15	
			1
		18 30	

In all only sixty *Leonids* were recorded, of which thirty-seven were noted by one observer and thirty-five by the other. In addition several sporadic meteors were also seen.

The increasing number of meteors between 16h 30m and 18h may possibly be due to the darkening of the sky as the Moon approached the horizon. It set at about 17h G.M.T.

At 17h 40m 1s a very brilliant meteor was seen by all four observers. It passed from *Leo* through a point midway between *Polaris* and *Capella*. It exhibited a bright train which nearly disappeared after two or three seconds. Then a portion of the train reappeared, brightened, and quickly changed form. After a little while the train bent in the middle and seemed to consist of two straight portions inclined at an obtuse angle to each other. By 17h 41m it had become indefinite and distorted into an irregular sinuous figure. At 17h 43m it seemed to hang from the star 22 (Hev.) *Camelopardali*, and had drifted a little towards *Polaris*. At this time its axis was situated at right angles to its former position. It eventually broke into two portions and faded away at 17h 46m 4s.

On the following night (November 15) a watch was again instituted, but up till midnight a fine drizzle prevailed, and the sky remained obstinately overcast until after 17h. At about 17h 30m the clouds began to part, and breaks occurred large enough to reveal whole constellations at a time, but no meteors were seen. These breaks were sufficiently open and numerous to

enable a definite opinion to be formed that the radiant then was very inactive. The sky again became overcast at $17^h\ 46^m$, and remained so until after sunrise.

Observations of the Leonids of 1899, *made at Durham Observatory.*

(*Communicated by Professor R. A. Sampson.*)

Preparations were made for a count of the expected shower upon a plan adapted from that of Professor E. C. Pickering's Harvard Circular. Maps and directions were given to between thirty and forty correspondents, here and at various places in Yorkshire, Durham, and Northumberland. The sky was divided into six segments, meeting at the radiant and passing through (1) *Regulus—ϵ Hydræ*, (2) *ϵ Hydræ—Pleiades*, (3) *Pleiades—Pole*, (4) *Pole—ζ Ursæ Majoris*, (5) *ζ Ursæ Majoris—β Leonis*, (6) *β Leonis—Regulus*. Each observer was to count the number of meteors appearing per minute in his own segment. It was hoped that thus the count might be pushed nearer the maximum, and perhaps some result of interest might appear as to whether the shower began on one side and died away on another. In the actual event the display was so meagre that the directions given were unsuitable, and little interest attaches to the observations except as an item of the average.

At the Observatory watch was kept on November 11, 12, 13, $16^h\ 30^m$—18^h; and November 14, 15, 11^h—18^h. On the 12th, 13th, and 14th it was densely clouded. Mr. F. C. H. Carpenter and myself watched on the 11th, and on the 14th and 15th we were joined by six of my students. We saw the following meteors, besides others which were not from the radiant in the Sickle. The paths of some of these were charted.

No.	Time.	h	m	Segment of Sky.	Note.
1	Nov. 11	17	31	5	b k g.
2	15	13	27	6	
	Between $13^h\ 27^m$ and $15^h\ 40^m$, five or six meteors seen, of which exact record was not taken.				
3	Nov. 15	16	3	2	
4		16	14	2	k or.
5		16	26	6	k y.
6		16	33	5	
7		16	35	3	
8		16	38	4	k bl.
9		16	41	6	k g.
10		16	42	6	k bl.

No.	Time.	Segment of Sky.	Note.
	h m		
11	Nov. 15 16 48	2	vb k (5 seconds), curved path.
12	16 51	4	k or.
13	17 6	2	k bl.
14	17 8	5	y.
15	17 10	3	y.
16	17 24	5	k bl.
17	17 25	2	bl.
18	17 27	2	
19	17 41	6	b k bl.
20	17 45	6	k y.
21	17 46	6	b or.
22	17 49	2	k bl.
23	17 50	3	b k g.
24	17 52	4	k or.
25	18 4	3	or.
26	18 8	5	bl.
27	18 17		

b = bright, k = streak, g = green, bl = blue, or = orange, y = yellow, v = very.

The numbers that appeared in successive quarters of an hour from $15^h\ 45^m$ to $18^h\ 15^m$ were respectively, 0, 2, 1, 5, 2, 3, 3, 2, 4, 2, or at the rate of 1 in $6\frac{1}{4}$ minutes. If for the sake of illustration we analyse these according to the segments in which they were seen, we get

Segment.	1	2	3	4	5	6	7	8	9	Totals.
1	...	...	...	...	...	...	...	...	...	0
2	...	...	...	1	1	2	...	1	...	5
3	2	...	1	...	1	...	...	1	1	6
4	...	...	1	1	...	...	...	1	...	3
5	...	...	1	...	1	1	...	...	1	4
6	...	1	2	...	...	...	2	1	...	6
Totals	2	1	5	2	3	3	2	4	2	24

It seems odd that none was seen in the first segment; apart from this the directions seem indiscriminate. But the numbers are altogether too small to carry any weight.

Durham: 1899 *November* 27.

Results of Observations of the Leonid Meteors of 1899 *as seen from the Royal Observatory, Edinburgh.* By George Clark, M.A.

(*Communicated by the Astronomer Royal for Scotland.*)

The sky in the neighbourhood of *Leo* was carefully watched during the whole of the night of Tuesday, November 14, and Wednesday, November 15, from 10.30 P.M. till 6.30 A.M. on each occasion. On both nights the sky was perfectly cloudless, but the fact that the gibbous Moon was riding high in the heavens made it difficult to observe the fainter meteors, and proved a serious drawback to photographic work.

The first *Leonid* on Tuesday was seen at $11^h\ 45^m$, and from then on till 16^h they occurred at long intervals, sometimes even half an hour elapsing without a single meteor being observed. As each appeared its apparent path was marked as accurately as possible on previously prepared maps. After 16^h it became evident that the shower was increasing, and the meteors gradually occurred more frequently till 17^h, when for a short time they appeared at average intervals of perhaps one minute.

Throughout the whole night the great majority of the paths radiated from the vicinity of the Sickle towards the southern half of a circle described round the radiant, very few meteors being observed to travel in a northerly direction. There was no difficulty whatever in discriminating between the *Leonids* and other meteors of a more sporadic nature which from time to time flashed across the sky. The radiant character of the former was very evident, and is distinctly brought out by the paths as drawn on the maps. Moreover, the *Leonids* shot forth with great swiftness, and were of an intense bluish-white colour, bearing indeed a marked resemblance to certain kinds of rockets. They had the same sort of bright bulb at the head, and left similar trails, some of which, even in the bright moonlight, persisted for several seconds. Some of these trails were very long, indicating that the meteors had passed "broad on" to the observer; while others were much foreshortened, owing to perspective. It was noticeable that these very short paths were almost all entirely within the bounds of the Sickle. The meteors seen between 17^h and $17^h\ 30^m$ seemed to be the most brilliant of the display; but as the Moon set at $17^h\ 19^m$ this was doubtless largely due to the consequent greater darkness of the sky. It did not occur to any of us that what we saw at this time could possibly represent the main stream of meteors, and hopes were confidently entertained that the grandest part of the display was still to come.

On Wednesday night, however, there was a considerable amount of haze; and at 14^h a damp, heavy fog came on, which lasted for about an hour. Owing to these circumstances it is more than likely that faint meteors entirely escaped observation;

indeed, during the whole watch of seven hours only two meteors of any description were seen, neither of them being a *Leonid*.

Arrangements were made by Mr. Heath for photographing the meteors by mounting two cameras—one of them for plates 18″ by 16″—on the 24-inch reflecting telescope. Between 17^h and 18^h on Tuesday, while the display was at its best, two plates were exposed; but unfortunately the attempt was not attended with any success.

Two charts were used on which to draw the apparent paths of the meteors observed. The first contains thirty-two meteors, and was in use on Tuesday night from 11^h till $16^h\ 30^m$; while the second contains forty-two meteors observed on the same night between $16^h\ 30^m$ and $18^h\ 45^m$. For the directions of several paths on each map I am indebted to Dr. Halm.

The method adopted in determining the apparent radiant point and the results obtained are briefly as follow:—

In the first place the arrows on each map were taken in pairs, such that the components of each pair were as nearly as possible at right angles to one another, no single arrow being made use of more than once. In this way twelve pairs were obtained on the first map and fourteen on the second. The exact position of the intersection of each pair was then determined, with the results exhibited in Table I.

TABLE I.

Map I. R.A. h	Decl. °	Map II. R.A. h	Decl. °
10·10	+24·0	9·92	+22·7
10·06	22·0	9·88	22·5
9·88	23·9	10·02	21·2
10·04	23·2	9·98	22·9
9·93	23·6	9·97	21·9
10·06	23·4	9·97	21·9
9·98	22·8	10·02	23·4
9·86	21·5	10·08	23·1
9·90	23·4	9·97	23·2
10·01	22·8	9·98	24·1
9·90	22·0	10·00	23·2
10·05	24·0	9·82	22·5
		9·96	22·0
		9·98	22·2

The mean of the results in Map I. is—

R.A. $9^h\ 58^m{\cdot}8 \pm 1^m{\cdot}4$; Decl. $+23°\ 3' \pm 15''{\cdot}3$

and in Map II.—

R.A. $9^h\ 57^m{\cdot}6 \pm 1^m{\cdot}0$; Decl. $+22°\ 37''{\cdot}2 \pm 12''{\cdot}4$;

while the mean of the results of both maps taken together is

$$\text{R.A. } 9^h\ 58^m{\cdot}2 \pm 0^m{\cdot}8\ ;\ \text{Decl. } +22^\circ\ 48'{\cdot}0 \pm 9'{\cdot}5.$$

To compute the amount of the zenith attraction, and the *true* position of the radiant point, it was necessary first to determine the zenith distance (Z) of the *apparent* radiant. This was computed (1) for the hour angle of the apparent radiant corresponding to the mean of the times (viz. $15^h\ 0^m$) at which the observations on the first map were made, and (2) for the similar hour angle at the mean of the times (viz. $17^h\ 37^m$) recorded on the second map. The amount of the zenith attraction was then obtained from Schiaparelli's formula

$$\tan \tfrac{1}{2}\phi = \frac{w-\omega}{w+\omega} \tan \tfrac{1}{2} Z,$$

where ϕ=zenith attraction ; w=accelerated velocity of meteors due to Earth's attraction ; ω=orbital velocity of meteors ; and Z=apparent zenith distance of radiant.

Finally, to determine the corrections to the R.A. and Decl. of the apparent radiant, we have

$$d\alpha = \phi \sin \eta \sec \delta\ ;\ d\delta = -\phi \cos \eta,$$

η being the parallactic angle.

The results thus deduced are arranged in order in the following table (II.) :—

TABLE II.

Map.	Z.	ϕ	$d\alpha$.	$d\delta$.	Corrected Position of Apparent Radiant. R.A.	Decl.	Weight.
	° ′	′	m	′	h m	° ′	
I.	49 25·8	20·2	+0·8	−16·6	9 59·0	+22 31·4	1
II.	34 8·5	13·5	+0·2	−13·2	9 58·4	+22 34·8	1·16

By combining the R.A.'s and declinations given in the last two columns according to their weights, the mean position of the *true* radiant point was found to be

R.A. $9^h\ 58^m{\cdot}6$; Decl. $+22^\circ\ 33'{\cdot}2$ (equinox of 1900·0).

Assuming this position for 1899 November, 14·73 G.M.T., the elements of the orbit have been computed for the period 33·25 years. But, as remarked by Dr. Rambaut in his paper reprinted from the *Proceedings of the Royal Society*, vol. lxv., there appears to be reason for believing that the most probable value of the period of revolution is at present about 33·49 years. The elements have therefore also been computed for this period, and the two sets of results are given in the first two columns of Table III. subjoined. For the sake of reference Dr. Rambaut's results for 1898 are given in column 3. Column 4 contains the elements for 1866 as established by the late Professor Adams, and column 5 the elements of the orbit of Comet I., 1866 (Temple's)

as computed by Dr. von Oppolzer. In each of these last two cases the elements have been brought up to the epoch 1900·0.

TABLE III.

		1 Edinburgh, 1899.	2	3 Rambaut, 1898.	4 Adams, 1866.	5 Comet I., 1866.
Period	P	33·25 yrs.	33·49 yrs.	33·49 yrs.	33·25 yrs.	33·17582
Mean Distance	a	10·34	10·39	10·39	10·34	10·3247
Angle of Eccentricity	ϕ	64°45′·0	64°48′·7	64°50′·0	64°47′·0	64°52′·8
Inclination	i	16°31′·3	16°31′·2	16°3′·0	16°45′·87	17°17′·95
Longitude of ☊	☊	52°39′·7	52°39′·7	53°2′·0	51°54′·97	51°53′·75
Longitude of Perihelion	π	56° 1′·7	56° 4′·2	58°40′·0	58°45′·17	42°52′·5
Perihelion Dist.	q	0·98789	0·98794	...	0·9885	0·97652
Time of Perihelion Passage	T	Nov. 12^d·257	Nov. 12^d·251	...	...	Jan. 11·13388

Royal Observatory, Blackford Hill, Edinburgh:
1899 *December* 7.

Erratum in Monthly Notices, vol. lx. page 18 (last line).

For 9^m to ·49^m·2, *read* 9^m·4 to 9^m·2.

CONTENTS.

PAGE

Fellows elected 81
Candidate proposed . . . . . *ib.*
Presents announced . . . . . *ib.*
G. H. Darwin, The theory of the Figure of the Earth carried to the second order of small quantities 82
Ernest W. Brown, Note on the values of the coefficients of the terms of the third order in the new Lunar Theory 124
F. A. Bellamy, On the proper motions of Berlin B, Nos. 5072 and 5073 . 125
James E. Keeler, New nebulæ discovered photographically with the Crossley reflector of the Lick Observatory 128
H. A. Howe, Nebulæ discovered at the Chamberlin Observatory, University Park, Colorado 129
H. A. Howe, Observations of Nebulæ made at the Chamberlin Observatory, University Park, Colorado 130
H. A. Howe, The Bruce micrometer of the Chamberlin Observatory . 140
William Ellis, On the relation between magnetic disturbance and the period of solar spot frequency 142
Royal Observatory, Greenwich, Mean areas and heliographic latitudes of Sun-spots in the year 1898, deduced from photographs taken at the Royal Observatory, Greenwich, at Dehra Dûn (India), and in Mauritius 157
Royal Observatory, Greenwich, Observations of the *Leonid* meteors of 1899 163
H. H. Turner, Observations of the *Leonids* of 1899, made at the University Observatory, Oxford 164
Radcliffe Observatory, Oxford, Observations of the *Leonids* of 1899 November 14 and 15 165
Durham Observatory, Observations of the *Leonids* of 1899 . . . 167
George Clark, Results of observations of the *Leonid* meteors of 1899, as seen from the Royal Observatory, Edinburgh 169
Erratum 172

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